Cooperative Positioning Algorithms for Estimating Inter-Vehicle Distance Using Multi-GNSS

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***Abstract*—In this paper, non ranging-based cooperative positioning algorithms based on GNSS measurements, including Absolute Position Differencing (APD), Single-Differencing (SD), and Double-Differencing (DD), are used to estimate the inter-vehicle distance (IVD). To reduce the uncertainty of IVD estimates, the maximum volume algorithm (MVA) was employed to determine the optimal geometric group composition of four satellites as a multi-GNSS system, namely GPS, GLONASS, Galileo, and BeiDou. Real-world experiments on two autonomous vehicles using SD-based and DD-based algorithms demonstrate that estimated IVD uncertainty is sub-centimetres, which is close to sensor-based solutions but at a lower cost.**

***Index Terms*—Inter-vehicle-distance (IVD), Absolute Position Differencing (APD), Single-differencing (SD), Double-differencing (DD), Multi-GNSS systems**

I. Introduction

The development and mass manufacturing of autonomous vehicles has the potential to change mobility and safety in transportation. As a key component of autonomous vehicles, advanced driver assistance systems play a critical role in increasing road safety, which significantly relies on inter-vehicle distance (IVD). Sensor-based technologies can accurately estimate the IVD by measuring the relative vehicle distances with, e.g., a Radio detection and ranging (Radar) and Light Detection Ranging (LiDAR) [1] or adopting a camera system [2]. However, high cost, poor performance in the adverse weather conditions, and limited perceptual fields remain major issues. The restricted perceptual range of these sensor-based approaches can be addressed via cooperative positioning algorithms [3]–[5]. In general, cooperative positioning algorithms are classified as either ranging-based or non ranging-based. For IVD estimation in ranging-based solutions, signal strength variations such as radio signal strength [6], Time of Arrival [7], round trip time [8] or Time Difference of Arrival [9] can be used. However, these approaches are often expensive since they require additional infrastructure and hardware to be implemented. The non ranging-based cooperative algorithms, on the other hand, provides the most cost-effective solutions by measuring the IVD directly from each vehicle's pseudorange information [3]–[5]. Since GNSS systems are undoubtedly low cost and can be employed independently in cooperative connected vehicles [3]–[5], several studies have been conducted as non-ranging cooperative positioning. Müller et al. [4] analyzed the use of GNSS double difference pseudoranges to estimate the relative position of two vehicles using two Bayesian filters. Yang et al. [10] proposed the weighted least squares pseudorange double difference algorithm for accurate IVD estimation in Dedicated Short Range Communications (DSRC) vehicular networks. Tahir et al. [5] provided a theoretical framework for measuring IVD using GNSS measurements exchanged between vehicles. Recently, Wang et al. [3] examined the performance of four non ranging-based cooperative algorithms for IVD estimation including Absolute Position Differencing (APD), Pseudorange Differencing (PD), Single Differencing (SD), and Double Differencing (DD) in static and dynamic experiments. Although the IVD estimation problem utilizing GNSS measurements have been extensively studied in the literature, they were limited to a single satellite system and did not take into account the Multi-Constellation Multi-Frequency system, despite the fact that these systems are becoming increasingly available to reach centimetre-level accuracy [11] and can increase the overall system robustness. In this paper, we extend the Multi-Constellation Multi-Frequency system with multiple satellites to increase the IVD accuracy and reliability. More in-depth, we studied the IVD estimation problem with real-world data coming from four satellites, including GPS, GLONASS, BeiDou, and Galileo, and we employed non ranging-based cooperative positioning algorithms that use sharing GNSS measurements, namely APD, SD and DD. Furthermore, the maximum volume algorithm (MVA), which is well-studied in [12], was used to determine the optimal configuration composition of four satellites with the lowest geometric dilution of precision (GDOP) to provide the highest IVD estimation accuracy.

This paper is built up as follows. Section II provides the mathematical formulation of the IVD estimation problem using the APD, SD, and DD algorithms. Section III describes the real-world experiment's configuration, including the study interval and the Lagrange interpolation for calculating the satellite positions. Section IV explains how to determine the optimal configuration of four satellites in various systems including one-, two-, three-, and four-systems of satellites using the MVA algorithm. Section V compares the estimated IVD among the best sets of satellites. Finally, conclusions are drawn in Section VI.

II. PROBLEM SET-UP

The GNSS raw measurements considered, denoted by ρ, are defined as the distance between a vehicle $V \in \{v_1, v_2, v_3, .., v_n\}$ and a satellite $S \in \{S_1, S_2, S_3, .., S_m\}$ at any time-step k, which are modeled as follows [3]:

$$\rho_V^S(k) = R_V^S(k) + t_V^S(k) + \varepsilon_c(k) + \varepsilon_u(k) \tag{1}$$

where $R_V^S(k) = ||P_S(k) - P_V(k)||$ is the true range between the vehicle V and the satellite S, $P_S(k) = [x_S(k), y_S(k), z_S(k)]^T$ is the position vector of the satellite S, $P_V(k) = [x_V(k), y_V(k), z_V(k)]^T$ is the position vector of the vehicle on the Earth-centered, Earth-fixed (ECEF) coordinate system, $t_V^S(k)$ is the time delay error between the receiver and the satellite, $\varepsilon_c(k)$ is the correlated uncertainty induced by the ephemeris and the atmosphere, and finally, $\varepsilon_u(k)$ denotes the uncorrelated uncertainty, which includes the multi-path error and the thermal noise.

A. Cooperative positioning algorithms

1) Absolute Position Differencing (APD): The GNSS receiver installed on each vehicle is able to compute an estimate of its absolute position vector in ECEF coordinates after acquiring and tracking the GNSS signal of at least four satellites. The absolute position differencing (APD) method calculates the distance between two vehicles at any time-step k denoted by $D_{ij}(k) = ||P_{v_1}(k) - P_{v_2}(k)||$, i.e.

$$D_{ij}(k) = \sqrt{(z_{v_2} - z_{v_1})^2 + (y_{v_2} - y_{v_1})^2 + (x_{v_2} - x_{v_1})^2} \tag{2}$$

where $(x_{v_1}, y_{v_1}, z_{v_1})$ and $(x_{v_2}, y_{v_2}, z_{v_2})$ are the ECEF coordinates of vehicle 1 and vehicle 2 obtained at time-step k from the GNSS, respectively. We assume here that the two autonomous vehicles equipped with the GNSS receivers additionally have a Real-time kinematic (RTK) system that calculates the distance between itself and the satellite which is broadcasting. Therefore, utilizing the RTK data, the estimated IVD by the APD approach is assumed to be the actual ground truth between the two vehicles.

2) Single-differencing (SD): Fig. 1 depicts the single differencing approach used for the IVD. The SD method estimates the IVD by subtracting the pseudorange measurements of two vehicles from the same satellite. This approach can eliminate both the clock imperfect synchronization between the vehicles as well as the atmospheric delay error. Given that the satellite S is sufficiently far from vehicles, the pseudorange measurements from each vehicle toward the satellite S are considered to be parallel (see Fig. 1) [3], [5]. More precisely, given (1) for two vehicles v_i and v_j, computing the difference we have:

$$\begin{aligned}\Delta\rho_{v_iv_j}^S(k) &= \rho_{v_i}^S(k) - \rho_{v_j}^S(k) = \\ &= \Delta R_{v_{i,j}}^S(k) + \Delta t_{v_{i,j}}(k) + \Delta\varepsilon_{v_{i,j}}^S(k)\end{aligned} \tag{3}$$

where $\Delta R_{v_{i,j}}^S(k)$ defines the difference between the true distance of vehicle v_i and vehicle v_j from the satellite S, $\Delta t_{v_{i,j}}(k)$ denotes the time delay error, and $\Delta\varepsilon_{v_{i,j}}^S(k)$ represents all the remaining uncertainties, usually dubbed *unusual error* [3], [5]. Due to the difference among the measured pseudoranges, the unusual error appears to be increasing [3]. Since the true distances between the vehicles and the satellites are much larger than the distance between the vehicles, we can estimate the $\Delta R_{v_{i,j}}^S(k)$ as follows [3], [5]:

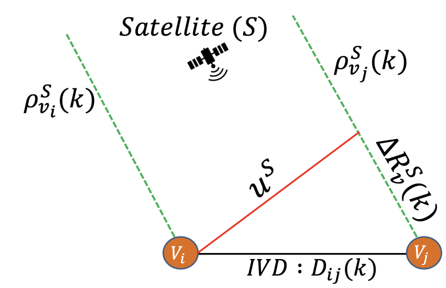

Fig. 1. Single-Differencing (SD-based) algorithm and triangle concept

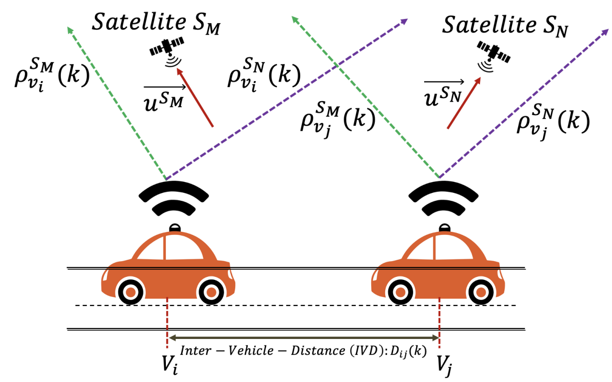

Fig. 2. DD-based IVD estimation algorithm

$$\Delta R_{v_{i,j}}^S(k) = [u^S]^T \overrightarrow{D_{ij}}(k) \tag{4}$$

where $\overrightarrow{D_{ij}}(k) = P_{v_i}(k) - P_{v_j}(k)$ whose norm is given by (2), $u^S = \frac{P_S(k) - P_{v_i}(k)}{||P_S(k) - P_{v_i}(k)||}$ is the line-of-sight unit vector from vehicle v_i to satellite S, $P_S(k)$ represent the position of the satellite S and $P_{v_i}(k)$ indicates the position of the reference vehicle v_i at time-step k (see Fig. 1 for reference). By considering N common visible satellites for the two vehicles and using (3), we can build the following measurement matrix

$$\begin{bmatrix}\Delta\rho_{v_iv_j}^1(k)\\ \Delta\rho_{v_iv_j}^2(k)\\ \vdots\\ \Delta\rho_{v_iv_j}^N(k)\end{bmatrix} \approx \begin{bmatrix}[u^1]^T & 1\\ [u^2]^T & 1\\ \vdots & \vdots\\ [u^N]^T & 1\end{bmatrix}\begin{bmatrix}\vec{D}_{ij}(k)\\ \Delta t_{v_{i,j}}(k)\end{bmatrix} \tag{5}$$

yielding the SD estimates [3], [5].

3) Double-differencing (DD): In the SD-based algorithm of (5), user clock offsets and uncorrelated errors are still present. To mitigate this uncertainties, we can utilize a new GNSS measurement and then computing the difference of the SD estimates obtained from two distinct satellites, say S_M and S_N. This is referred to as double-differencing (DD) and depicted in Fig. 2. The DD-based algorithm assumes that both vehicles can track satellites S_M and S_N at the same time.

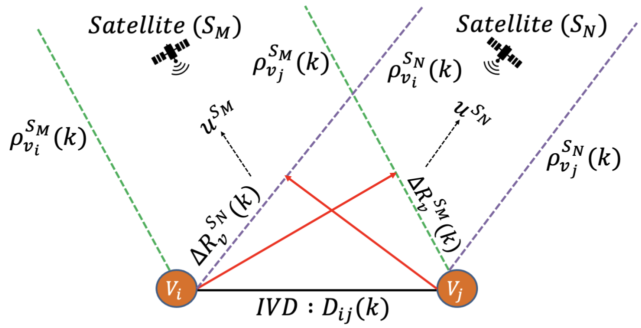

Fig. 3. DD-based IVD estimation algorithm and triangle concept

Hence, we first apply an SD-based algorithm to each vehicle toward the satellites S_M and S_N, denoted by $\Delta\rho_{v_i v_j}^{S_M}(k)$ and $\Delta\rho_{v_i v_j}^{S_N}(k)$, respectively, which are obtained from (3). Then, the difference of such quantities is obtained as:

$$\begin{aligned}\nabla\Delta\rho_{v_i v_j}^{S_M S_N}(k) &= \Delta\rho_{v_i v_j}^{S_M}(k) - \Delta\rho_{v_i v_j}^{S_N}(k) = \\ &= \Delta R_{v_i v_j}^{S_M S_N}(k) + \Delta\varepsilon_{v_i v_j}^{S_M S_N}(k)\end{aligned} \tag{6}$$

where $\Delta R_{v_i v_j}^{S_M S_N}(k) = \Delta R_{v_{i,j}}^{S_M}(k) - \Delta R_{v_{i,j}}^{S_N}(k)$ and $\Delta\varepsilon_{v_i v_j}^{S_M S_N}(k) = \Delta\varepsilon_{v_{i,j}}^{S_M}(k) - \Delta\varepsilon_{v_{i,j}}^{S_N}(k)$. We can then estimate $\Delta R_{v_i v_j}^{S_M S_N}(k)$ using the same trigonometric idea of SD, that is depicted in Fig. 3 [3]–[5].

$$\Delta R_{v_i v_j}^{S_M S_N}(k) = [u^{S_M} - u^{S_N}]\overrightarrow{D}_{ij}(k) \tag{7}$$

where u^{S_M} and u^{S_N} are computed as in (4). Using (6) is then possible to calculate the distance and the relative positions of two vehicles. Indeed, using the satellite M as reference, the solution to the DD-based algorithm according to Fig. 3 is given by the matrix form [3], [4]:

$$\begin{bmatrix}\nabla\Delta\rho_{v_i v_j}^{S_1 S_M}(k) \\ \nabla\Delta\rho_{v_i v_j}^{S_2 S_M}(k) \\ \vdots \\ \nabla\Delta\rho_{v_i v_j}^{S_N S_M}(k)\end{bmatrix} \approx \begin{bmatrix}[u^1 - u^M]^T \\ [u^2 - u^M]^T \\ \vdots \\ [u^N - u^M]^T\end{bmatrix} \overrightarrow{D}_{ij}(k) \tag{8}$$

Notice that the IVD vector $\overrightarrow{D}_{ij}(k)$ is projected in the direction of the difference satellite unitary vectors $\overrightarrow{u}^{S_{MN}} = \overrightarrow{u}^{S_M} - \overrightarrow{u}^{S_N}$ for each DD measurement $\nabla\Delta\rho_{v_i v_j}^{S_{MN}}(k)$. Assuming four satellites, say S_M, S_N, S_O, and S_P, and considering S_M as the reference satellite, the following system of linear equations derived from (8) can be obtained [4]:

$$\begin{aligned}\begin{bmatrix}\nabla\Delta\rho_{v_i v_j}^{S_{MN}} \\ \nabla\Delta\rho_{v_i v_j}^{S_{MO}} \\ \nabla\Delta\rho_{v_i v_j}^{S_{MP}}\end{bmatrix} &= \begin{bmatrix}u_x^{S_{MN}} & u_y^{S_{MN}} & u_z^{S_{MN}} \\ u_x^{S_{MO}} & u_y^{S_{MO}} & u_z^{S_{MO}} \\ u_x^{S_{MP}} & u_y^{S_{MP}} & u_z^{S_{MP}}\end{bmatrix}\begin{bmatrix}D_x \\ D_y \\ D_z\end{bmatrix} =, \\ &= H_{uu}\overrightarrow{D}_{ij}(k)\end{aligned} \tag{9}$$

where

$$\begin{aligned}\overrightarrow{u}^{S_{qr}} &= \frac{S_q(k) - P_{v_i}(k)}{||S_q(k) - P_{v_i}(k)||} - \frac{S_r(k) - P_{v_i}(k)}{||S_r(k) - P_{v_i}(k)||} = \\ &= \begin{bmatrix}u_x^{S_q} \\ u_y^{S_q} \\ u_z^{S_q}\end{bmatrix} - \begin{bmatrix}u_x^{S_r} \\ u_y^{S_r} \\ u_z^{S_r}\end{bmatrix},\end{aligned}$$

where S_q and S_r, $q, r \in \{M, N, O, P\}$ are the satellite positions and P_{v_i} is position of the vehicle v_i, all evaluated at the time-step k. Notice that 4 is the minimum number of satellites needed to have a solution of the DD-based algorithm, i.e., H_{uu} (known as the *geometry matrix*) should be non singular. Usually, if more than 4 satellites are available, a more precise and effective Least Squares solution is adopted.

B. Geometric dilution of precision (GDOP)

To compute the GDOP as a figure of merit for the reachable uncertainty [13], we first assume that all the GNSS measurements are zero-mean and with equal variance σ_ρ^2, which yields:

$$C_x = \sigma_\rho^2 (H_{uu}^T H_{uu})^{-1} = \begin{bmatrix}D_{11} & D_{12} & D_{13} & D_{14} \\ D_{21} & D_{22} & D_{23} & D_{24} \\ D_{31} & D_{32} & D_{33} & D_{34} \\ D_{41} & D_{42} & D_{43} & D_{44}\end{bmatrix} \tag{10}$$

Then GDOP is finally given by:

$$GDOP = \sqrt{D_{11} + D_{22} + D_{33} + D_{44}}. \tag{11}$$

III. Optimal satellite selection procedure

The maximum volume algorithm (MVA), a four-steps heuristic method for picking four satellites based on the form of a tetrahedron, was proposed by Kihara et al. [12] as follows:

- **Step.1**: Select the visible satellite S_1 with the largest elevation angle relative to the position of the receiver.
- **Step.2**: Choose the visible satellite S_2 having the angle to S_1, i.e., $\theta_{S_1 S_2}$, close to $109.47°$.
- **Step.3**: Pick the visible satellite S_3 that maximizes the volume of the tetrahedron:

$$\begin{aligned}V_A = \frac{1 - a_3}{6}\Big[&\sqrt{2(1 - a_2)(1 + a_3)(1 - a_2 a_3 - b_2 b_3)} + \\ &+ |b_2 c_3|\Big],\end{aligned} \tag{12}$$

where

$$a_2 = \cos\theta_{S_1 S_2}, b_2 = \sin\theta_{S_1 S_2}, a_3 = \cos\theta_{S_1 S_3},$$
$$b_3 = \frac{\cos\theta_{S_2 S_3} - a_2 a_3}{b_2}, c_3 = \pm\sqrt{1 - a_3^2 - b_3^2}.$$

Notice that the tetrahedron is formed by S_1, S_2, S_3.

- **Step.4**: Select the satellite S_4 from the remaining visible satellites so that it maximizes the volume of the tetrahedron

$$V_B = \frac{1}{6} det(S) \tag{13}$$

where S is the matrix that contains the line-of-sight vectors corresponding to S_1, S_2, S_3, and S_4.

IV. Real-world Experiment Configuration

We conducted a real-world experimental test for two autonomous vehicles collecting pseudorange data from different satellites. We first considered all 1500 available epochs for the two vehicles in our case study, while considering two known satellite positions in the first and last epochs taking from the NASA service data on April 26, 2022 at 12:55:00

and 13:00:00 UTC time [14]. Then, as shown in Fig. 4, we employed the Lagrange interpolation approach to compute satellite positions epoch-by-epoch across the whole study interval. For this purpose, we consider $L_s \in \{l_0, l_1, l_2, ..., l_n\}$ be the values of the satellite locations, i.e., $L_s = [x_s, y_s, z_s]$, in times at $t \in \{t_0, t_1, t_2, ..., t_n\}$. The first and final known satellite positions taken from NASA service data are then used as inputs for the Lagrange method to calculate the approximate value of l, denoted by $L(t)$ at any time of t as follows [15]:

$$L(t) = a_0 l_0 + a_1 l_1 + a_2 l_2 + ... + a_n l_n = \sum_{i=0}^{n} a_i l_i \tag{14}$$

where:

$$a_i = \frac{(t-t_0)(t-t_1)...(t-t_{i-1})(t-t_{i+1})...(t-t_n)}{(t_i-t_0)(t_i-t_1)...(t_i-t_{i-1})(t_i-t_{i+1})...(t_i-t_n)} \tag{15}$$

Now, by substituting t in Eq. 14 with $\{t_0$, t_1, t$_2$,..., $t_n\}$, we obtain:

$$L(t_0) = l_0, L(t_1) = l_1, ..., L(t_n) = l_n \tag{16}$$

According to [15], while dealing with Lagrange interpolation for computing satellite positions, we typically have an error in the beginning and ending points of the interpolation. Thus, we considered a validity interval for dealing with Lagrange interpolation errors in our satellite positioning by ignoring the start and final 10% of the data-set, as proposed in [15]. To evaluate the performances, the root mean squared error (RMSE) of the IVD is computed as:

$$RMSE = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left[D_{ij}(k) - \hat{D}_{ij}(k)\right]^2} \tag{17}$$

where $D_{ij}(k)$ is the true distance between the two vehicles v_i and v_j at time-step k and $\hat{D}_{ij}(k)$ is the estimated IVD at time-step k. N is the number of total epochs during the interval period, which is $N = 250$. It has to be recalled that the ground-truth IVD $D_{ij}(k)$, which is computed as in (1), is the one returned by the APD with the RTK system. In all the following experimental results, this value is over the entire study interval is 3.35 m.

A. One-system of satellites

The total number of common visible satellites in our case study is 26 (see Fig. 5). In the first set of study, we consider only one group of satellites, say GPS, GLONASS, Galileo, or BeiDou. To select the four satellites, we used the MVA algorithm previously depicted. For example, Table I subsumes the optimal choice for the GPS satellite system with the four steps of the MVA exemplified. In the end, among the 8 available satellites, G18, G05, G16 and G31 were chosen in order. Table II subsumes the MVA results obtained for all the available satellite systems, i.e., including GLONASS, Galileo, and BeiDou. According to Table II and Fig. 6, the Galileo satellite system has the lowest RMSE and the closest IVD for both the SD and DD algorithms to the ground truth APD. Notice that, as mentioned previously, the DD gains the highest performance. Moreover, the poor performance of the GLONASS confirms our first statement that we cannot rely only on one system of satellites for measuring the IVD due to possible satellite visibility limitations.

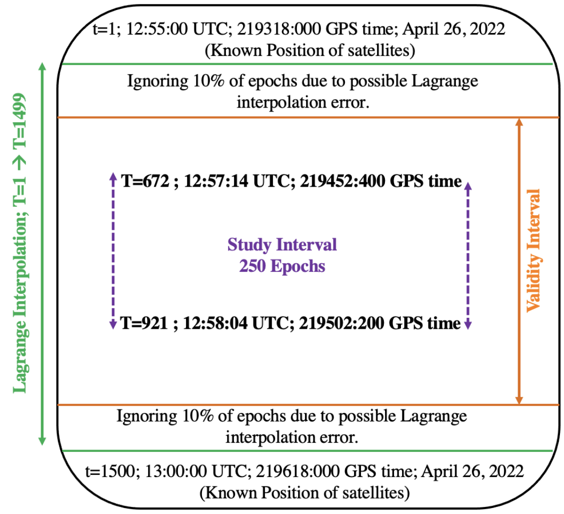

Fig. 4. Structure of the study interval

TABLE I
OPTIMAL CONFIGURATION OF FOUR SATELLITES FROM ONLY GPS

One-system of satellites (GPS)				
Satellite	**Step 1** Elevation angle (deg)	**Step 2** Angle to S_1 (deg)	**Step 3**: V_3	**Step 4**: V_4
S_2:**G05**	17.221	**105.23**	———	———
S_3:**G16**	46.705	17.327	**0.31456**	———
S_1:**G18**	**75.032**	——	———	———
G23	23.833	111.26	0.10962	0.14394
G26	74.94	19.134	0.12668	0.016652
G27	20.485	15.836	0.29532	0.080569
G29	32.762	136.56	0.2463	0.022658
S_4:**G31**	21.306	57.955	0.047787	**0.168999**

System	PRN Code	System	PRN Code	System	PRN Code	System	PRN Code
GPS (USA)	G05	Galileo (EU)	E01	BeiDou (China)	C05	GLONASS (Russia)	R9
	G16		E12		C13		
	G18		E24		C24		R15
	G23		E26		C26		
	G26		E31		C29		R18
	G27		E33		C35		
	G29				C44		R19
	G31				C45		

Fig. 5. List of common visible satellites for the vehicles v_1 and v_2

B. Two-system of satellites

There are six scenarios to combine two satellite systems to find the optimal geometry configuration of four satellites, including GPS-GLONASS, GPS-Galileo, GPS-BeiDou, Galileo-BeiDou, Galileo-GLONASS and BeiDou-GLONASS. As for the one-system of satellites case, we used the MVA algorithm to determine the optimal geometry of configuration in those six cases. Table III provides a comparative analysis of the performance for all groups from the two-system of satellites.

TABLE II
AVERAGE ESTIMATED IVD IN METERS FOR ONE-SYSTEM OF SATELLITES

System of Satellite	**Optimal Configuration**	APD	DD	SD	RMSE
GPS	G05,G16,G18,G31	3.35	2.7987	2.8168	1.5049
GLONASS	R09,R15,R18,R19	3.35	28.326	28.326	25.489
Galileo	**E33,E31,E24,E26**	**3.35**	**3.294**	3.3294	**0.2761**
BeiDou	C35,C45,C13,C24	3.35	3.9477	3.9477	1.8117

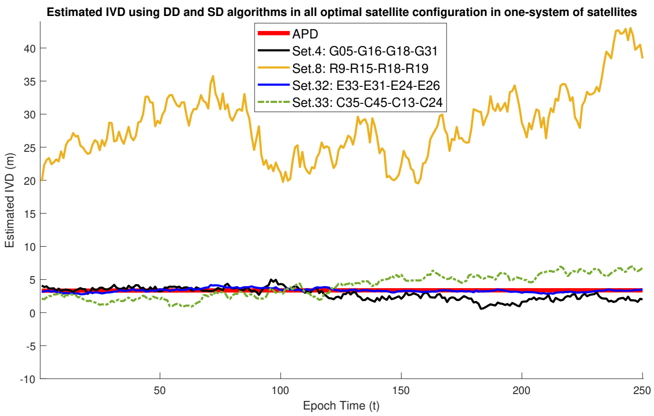

Fig. 6. Estimated IVD by SD- and DD-based algorithms in one-system of satellites

C. Three-system of satellites

There are three different satellite groups for the three-system of satellites, including GPS-Galileo-BeiDou, GPS-Galileo-GLONASS, and Galileo-BeiDou-GLONASS. Table IV summarizes the analysis performance for these cases.

D. Four-system of satellites

Finally, Table V shows that the optimal group of satellites when combining all the sources is C35, E24, G5, and R15.

V. Results and Discussion

From the presented analysis, we first notice that basically there is no difference between SD and DD when the number of satellite systems increases. In other words, the systematic effects affecting the pseudorange measurements appear to be cancelled out. This is somehow expected since every satellite systems suffer from different systematic uncertainties in the measurements. As a second result, we may notice that mixing together different satellite systems has indeed some benefits, as depicted in Fig. 7. It may be noted that, even accounting for system of satellites returning quite poor results (see the GLONASS results in Table II), the combination with other sources turns out to be a winning combination, as reported in Fig. 7 and in the first row of Table V. In other words, combining together multiple satellite systems surely increases the estimation robustness (we are not constrained to a single source of data) and, on the other hand, reaches similar target uncertainties. For what concerns the robustness and the favorable satellite configuration when multiple systems are adopted, we finally report in Fig. 8 the GDOP values as a function of

TABLE III
AVERAGE ESTIMATED IVD IN METERS FOR TWO-SYSTEM OF SATELLITES

System of Satellite	**Optimal Configuration**	APD	DD	SD	RMSE
GPS GLONASS	G18,G5,R9,R15	3.35	11.775	11.775	8.5636
GPS Galileo	G18,G5,E24,E12	3.35	4.2927	4.2927	1.1764
GPS BeiDou	**C35,G5,C24,C45**	**3.35**	**2.9276**	2.9276	**1.4279**
BeiDou Galileo	C35,C45,C13,C24	3.35	31.623	31.623	32.413
Galileo GLONASS	R18,R15,E24,E26	3.35	4.4782	4.4782	2.3952
BeiDou GLONASS	C35,R15,C13,C24	3.35	3.7108	3.7111	1.6596

TABLE IV
AVERAGE ESTIMATED IVD IN METERS FOR THREE-SYSTEM OF SATELLITES

System of Satellite	**Optimal Configuration**	APD	DD	SD	RMSE
GPS Galileo BeiDou	**C35,E1,G5,G27**	**3.35**	**2.8736**	2.8736	**0.6776**
GPS Galileo GLONASS	G18,G23,E24,G5	3.35	5.5057	5.5059	2.2201
BeiDou Galileo GLONASS	C35,E1,E24,C29	3.35	31.632	31.623	32.413

the epoch time (i.e., the time step k). As clearly depicted, the GDOP values decreases (hence, less ensuing uncertainty) from one-system of satellite to four-system of satellites. It should be noted that, while the performance of the one-system and four-system of satellites in our case study is closely comparable, the four-system of satellites has the lowest GDOP, as shown in Fig. 8. Furthermore, we may notice that even when the GDOP configuration worsen as a function of time (upper left picture of Fig. 8), this effect is highly mitigated (actually, inverted) when multiple systems are considered at once, thus further verifying the main message of this paper: using multiple satellite systems benefit the uncertainty related to the inter-vehicle distance.

As a final comparison, let us consider the absolute error, i.e., $\overline{\Delta d} = |D_{ij}(k) - \hat{D}_{ij}(k)|$, from the camera-based solution proposed in [16]. With such a solution, the estimated IVD in the best case of a partly occluded environment or in a fully occluded situation is 6.2 cm and 18.9 cm, respectively, which is comparable to the analysis proposed here but at a higher computational cost.

VI. Conclusion

This study proposed and experimentally validated that the use of a multi-GNSS system using either the Single-Differencing (SD) or the Double-Differencing (DD) algorithms can provide lower uncertainties in the determination of the inter-vehicle distance. Indeed, it has been shown with

TABLE V
AVERAGE ESTIMATED IVD IN METERS FOR FOUR-SYSTEM OF SATELLITES

System of Satellite	**Optimal Configuration**	APD	DD	SD	RMSE
BeiDou Galileo GLONASS GPS	**C35,E24,G5,R15**	**3.35**	**3.47201**	3.47312	**1.035**
GPS Galileo GLONASS BeiDou	C35,E1,G5,R19	3.35	4.6550	4.6555	2.4007
GLONASS Galileo GPS BeiDou	R18,C13,G23,E24	3.35	5.5900	5.5999	2.2893
GPS Galileo BeiDou GLONASS	C35,E1,G5,G27	3.35	2.8736	2.8736	0.6776

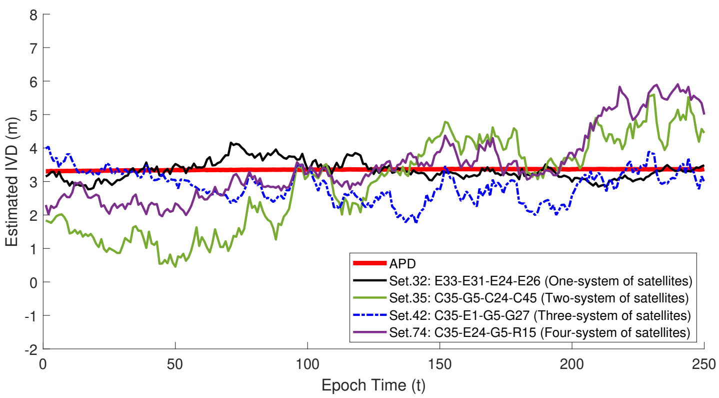

Fig. 7. Estimated IVD by the DD-based algorithm in the best configuration of satellites

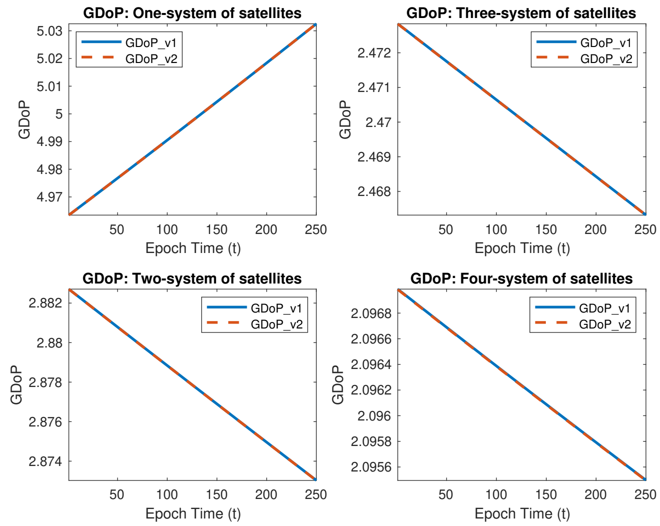

Fig. 8. GDOP in the optimal configuration of satellites

experimental evidence that relying on multiple satellite systems benefits the reachable uncertainty (as expressed with the GDOP) and the robustness. Moreover, it also have computation time reduction since both the SD and DD behaves similarly. In future research, we will focus on the application of the method on multiple vehicles (i.e., more than two) and in the improvement of the algorithmic solution provided by the SD or DD approaches.

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