

# Matrix-Free Gauss-Newton Method for Large-Scale Electrical Impedance Tomography

Jian Deng<sup>ID</sup>, Babu Linkoon P. Meenaketan<sup>ID</sup>, Dries Braeken, Yoke Chin Chai<sup>ID</sup>, Tom Torfs<sup>ID</sup>, Jan De Beenhouwer<sup>ID</sup>, *Senior Member, IEEE*, and Jan Sijbers<sup>ID</sup>, *Senior Member, IEEE*

**Abstract**—Electrical impedance tomography (EIT) is a non-invasive imaging technique that reconstructs the internal conductivity distribution of a target object, with applications spanning medical imaging, geophysics, and industrial process monitoring. While the iterative Gauss-Newton (GN) method is widely adopted for its superior reconstruction accuracy, conventional implementations suffer from prohibitive computational and memory demands in large-scale EIT problems. To overcome these limitations, this study introduces the concept of a single electrode model (SEM), which streamlines the calculation of the Jacobian matrix. Building upon SEM, we propose a computationally efficient implementation of the matrix-free Gauss-Newton (MF-GN) method for EIT, which eliminates the need for explicit storage and manipulation of large Jacobian matrices. The proposed MF-GN method attains reconstruction accuracy comparable to that of the conventional GN method while reducing memory usage by orders of magnitude and substantially accelerating computation for large-scale systems. Numerical experiments and real-data validations demonstrate its efficacy, underscoring the potential of MF-GN for solving large-scale EIT problems.

**Index Terms**—Electrical impedance tomography, Gauss-Newton method, large-scale problem, matrix-free method.

## I. INTRODUCTION

**E**LECTRICAL Impedance Tomography (EIT) is an imaging technique that reconstructs the conductivity distribution within a target region by analyzing boundary voltage and current measurements. Electrodes placed around the target region inject small electrical currents, and the resulting boundary voltages are recorded. Using these current-to-voltage measurements, an inverse problem is solved to estimate the internal conductivity distribution, typically by minimizing the difference between measured voltages and those simulated based on a forward model. Due to its non-ionizing, low-cost, and portable nature, EIT has found applications across a wide range of fields, including medical imaging (e.g., lung function monitoring, brain

imaging, and cancer detection) [1], in-vitro imaging [2], geophysical exploration [3], and industrial process monitoring [4]. Despite these advantages, the ill-posed and nonlinear nature of the EIT inverse problem poses significant challenges to achieving high-resolution and accurate imaging results.

Traditional EIT imaging methods encompass a variety of approaches, including the back-projection method [5] and the D-bar method [6]. The backprojection method offers simplicity and computational efficiency, but is limited in resolution and robustness to noise. The D-bar method, which is based on complex geometrical optics, provides direct reconstructions and is computationally efficient for certain scenarios. However, both methods often struggle with handling high levels of noise or providing high spatial resolution. These limitations have driven the development of iterative reconstruction techniques, such as gradient-based methods and the Gauss-Newton (GN) method, which iteratively refine the conductivity distribution to minimize an objective function that quantifies the discrepancy between observed and simulated voltage data.

Gradient-based methods, such as Conjugate Gradient (CG) [7], ADAM [8], and Limited-memory BFGS (L-BFGS) [9], compute the first-order derivative (gradient) of the objective function with respect to the conductivity model using an adjoint method [10], without explicitly calculating the Jacobian matrix. Gradient-based methods are memory efficient, making them well-suited for large-scale problems. In contrast, the GN method [11] explicitly employs the Jacobian matrix to approximate the second-order derivative (Hessian matrix) of the objective function, resulting in faster convergence and more accurate reconstructions compared to gradient-based methods. However, the computational and memory demands of the GN method become prohibitive in large-scale EIT applications. This limitation primarily arises from the need to explicitly compute and store the Jacobian matrix, as well as to solve the large linear system, both of which scale poorly with problem size. This issue is particularly severe in applications such as three-dimensional (3D) EIT of the brain, breast, and other biological tissues using hundreds of electrodes [12], [13], where the Jacobian matrix alone can require hundreds of gigabytes (GB) of memory.

In recent years, Deep Learning (DL) has significantly advanced EIT [14], [15], with methods broadly categorized into data-driven and model-driven approaches. Within the data-driven category, direct mapping is the most common method, utilizing Neural Networks (NNs) to learn a supervised relationship between boundary measurements and conductivity

Received 13 October 2025; revised 19 February 2026; accepted 21 March 2026. Date of publication 23 April 2026; date of current version 25 May 2026. The associate editor coordinating the review of this article and approving it for publication was Prof. Andreas Hauptmann. (Corresponding author: Babu Linkoon P. Meenaketan.)

Jian Deng, Jan De Beenhouwer, and Jan Sijbers are with the Imec-Vision Lab and  $\mu$ NEURO Research Centre of Excellence, University of Antwerp, 2610 Antwerpen, Belgium (e-mail: Jian.Deng@uantwerpen.be; jan.debeenhouwer@uantwerpen.be; jan.sijbers@uantwerpen.be).

Babu Linkoon P. Meenaketan, Dries Braeken, Yoke Chin Chai, and Tom Torfs are with the IMEC, 3001 Leuven, Belgium (e-mail: babulinkoon.meenaketan@imec.be; dries.braeken@imec.be; Yoke.Chin.Chai@imec.be; Tom.Torfs@imec.be).

Digital Object Identifier 10.1109/TCI.2026.3687033

distributions [16], [17], [18], [19]. However, these approaches often operate as a “closed box,” lacking physical interpretability and failing to guarantee that the results satisfy the underlying governing equations. Furthermore, they are highly sensitive to noise and domain changes. Another data-driven approach is the feature-based method, which incorporates multi-modality information to reduce ill-posedness. These methods use NNs to extract structural or anatomical features (e.g., organ boundaries) from other modalities, such as X-ray image [20], and integrate them as priors into the EIT framework. Despite their potential, both data-driven categories suffer from the difficulty of acquiring sufficient, high-quality training data for most EIT applications. Alternatively, model-driven methods, such as NN-regularized approaches, introduce implicit regularization by re-parameterizing the conductivity model with NNs without requiring training data [21], [22], [23], [24]. While these methods improve reconstruction accuracy, they typically require extensive trial-and-error in tuning hyperparameters due to a lack of theoretical guidance. This process is computationally prohibitive for large-scale EIT problems.

To mitigate the trade-off between reconstruction accuracy and computational cost encountered in existing EIT methods for large-scale applications, we propose integrating matrix-free (MF) computation techniques into the GN method. Originally developed for iteratively solving large linear systems where the system matrix is too large to be stored explicitly, the MF approaches eliminates the need to explicitly form or store the system matrix by computing matrix-vector products on-the-fly during each iteration [25], [26]. Such approaches have been successfully applied in medical imaging, including X-ray computed tomography [27] and magnetic resonance imaging [28], where the matrix-vector products can be computed efficiently. However, directly applying the MF approaches to the GN method in the context of EIT introduces significant challenges due to the high computational cost of repeatedly constructing the Jacobian matrix (explained in the Section II-D). To overcome this issue, this paper also proposes an efficient strategy for Jacobian matrix computation. The resulting matrix-free GN method (MF-GN) effectively mitigates both memory and computational bottlenecks of the conventional GN method without sacrificing the reconstruction accuracy, enabling high-fidelity imaging in large-scale EIT applications,

## II. METHODS

### A. Forward Problem

The forward problem of EIT can be modeled using either the Point Electrode Model (PEM) [29] or the Complete Electrode Model (CEM) [30]. The key distinction between these models lies in their treatment of electrode effects. The CEM incorporates the electrodes’ geometry and contact impedance, providing a more accurate representation of the physical measurement process. In contrast, the PEM simplifies the model by neglecting these effects, prioritizing computational efficiency. This simplification makes the PEM suitable for scenarios where the size of electrodes is negligible relative to that of the computational domain or in time-difference EIT applications, where electrode

effects are mitigated by subtracting measurements taken at different time points.

Mathematically, the PEM can be viewed as a special case of the CEM, where both electrode size and contact impedance are assumed to be zero. The CEM problem can be numerically solved using the Finite Element Method (FEM) [31]. Consider a bounded domain discretized into  $N$  vertices and  $M$  finite elements, with  $L$  electrodes attached to its boundary. Assuming Neumann boundary conditions, the FEM formulation of the CEM is expressed as [32], [33]:

$$\mathbf{A} \begin{bmatrix} \mathbf{u} \\ \hat{\mathbf{u}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{j} \end{bmatrix}, \quad (1)$$

where  $\mathbf{A} \in \mathbb{C}^{(N+L) \times (N+L)}$  is the system matrix, the vector  $\mathbf{u} \in \mathbb{C}^{N \times 1}$  represents the nodal potentials at the mesh vertices, while  $\hat{\mathbf{u}} \in \mathbb{C}^{L \times 1}$  contains the electric potentials on the electrodes. The vector  $\mathbf{j} = \{j_i\}_{i=1}^L \in \mathbb{C}^L$  specifies the net current injected through each electrode.

In (1), the system matrix  $\mathbf{A}$  is determined by the spatial distribution of the electrical conductivity, the finite element mesh topology, the electrode configuration, and the electrode–medium contact impedance. Notably,  $\mathbf{A}$  is independent of the applied current injection pattern. Initially,  $\mathbf{A}$  is rank-deficient due to the gauge freedom inherent in electric potential. However, this deficiency can be resolved by fixing a reference potential (e.g., grounding a node or electrode), after which  $\mathbf{A}$  becomes full rank, ensuring the existence and uniqueness of the potential distribution  $[\mathbf{u}; \hat{\mathbf{u}}]$ .

From a physical standpoint, the current injection vector  $\mathbf{j}$  must satisfy the law of conservation of electric charge:

$$\sum_{i=1}^L j_i = 0. \quad (2)$$

Once the system in (1) is solved for a given current injection, the voltage measurements (potential difference between two electrodes)  $\mathbf{v} \in \mathbb{R}^q$  can be computed as:

$$\mathbf{v} = \mathbf{D}\hat{\mathbf{u}}, \quad (3)$$

where  $\mathbf{D} \in \mathbb{R}^{q \times L}$  is the measurement operator matrix, and  $q$  denotes the number of voltage measurements during the current injection. Each row of  $\mathbf{D}$  contains exactly one entry of 1 and one entry of  $-1$ , corresponding to a specific pair of electrodes used for voltage measurement; all other entries are zero.

### B. Single Electrode Model

As indicated in (1), the potential distribution  $[\mathbf{u}; \hat{\mathbf{u}}]$  is linearly dependent on the current injection  $\mathbf{j}$ . Leveraging this linearity, we introduce the concept of the Single Electrode Model (SEM), in which a unit current is injected through a single electrode, with no current exiting elsewhere. In this model, the element of  $\mathbf{j}$  corresponding to the injection electrode is set to 1, while all other elements are set to 0. For a system with  $L$  electrodes, there are thus  $L$  SEM problems to be solved. Although these SEM problems are not physically realizable (since they violate current conservation), they remain mathematically solvable as

the square system matrix  $\mathbf{A}$  becomes full-rank once a ground point is specified.

By solving these  $L$  SEM problems using a common ground point and storing the resulting potential distributions, the potential distribution for any current injection satisfying the current conservation law (2) can be efficiently reconstructed through linear combinations of the SEM solutions.

As an example, in a system with  $L$  electrodes, there are  $C_L^2$  possible two-electrode current injections, where current flows into one electrode and exits through another. Conventionally, obtaining the potential distribution corresponding to these  $C_L^2$  patterns requires solving (1) individually for each current injection satisfying the current conservation law (2). To improve computational efficiency, the LU decomposition method is commonly employed, wherein the system matrix  $\mathbf{A}$  is factorized once into lower and upper triangular matrices  $\mathbf{L}$  and  $\mathbf{U}$ , and the following system is solved for each current injection:

$$\mathbf{L}\mathbf{x} = \begin{bmatrix} 0 \\ \mathbf{j} \end{bmatrix}, \quad \mathbf{U} \begin{bmatrix} \mathbf{u} \\ \hat{\mathbf{u}} \end{bmatrix} = \mathbf{x}. \quad (4)$$

Alternatively, by leveraging the SEM approach, the potential fields corresponding to all  $C_L^2$  two-electrode current injections can be efficiently constructed through linear combinations of only  $L$  precomputed SEM solutions. This approach requires solving (4) only  $L$  times. Specifically, the potential distribution corresponding to current flowing into electrode  $a$  and exiting through electrode  $b$  can be expressed as:

$$\begin{bmatrix} \mathbf{u}_{ab} \\ \hat{\mathbf{u}}_{ab} \end{bmatrix} = \begin{bmatrix} \mathbf{u}_a \\ \hat{\mathbf{u}}_a \end{bmatrix} - \begin{bmatrix} \mathbf{u}_b \\ \hat{\mathbf{u}}_b \end{bmatrix}, \quad (5)$$

where  $[\mathbf{u}_{ab}, \hat{\mathbf{u}}_{ab}]^\top$  denotes the potential distribution resulting from a current injected at the  $a$ -th electrode and extracted at the  $b$ -th electrode, and  $[\mathbf{u}_a, \hat{\mathbf{u}}_a]^\top$  and  $[\mathbf{u}_b, \hat{\mathbf{u}}_b]^\top$  are the potential distributions obtained from the SEM solutions corresponding to unit current injections at electrodes  $a$  and  $b$ , respectively.

Similarly, the corresponding electric fields are given by:

$$\mathbf{e}_{ab} = \mathbf{e}_a - \mathbf{e}_b, \quad (6)$$

where  $\mathbf{e}_{ab} = \nabla \mathbf{u}_{ab}$ ,  $\mathbf{e}_a = \nabla \mathbf{u}_a$ , and  $\mathbf{e}_b = \nabla \mathbf{u}_b$  represent the spatial gradients of the corresponding potential fields.

It is important to note that for a system with  $L$  electrodes, the number of possible current injections satisfying the current conservation law is theoretically infinite. However, the number of linearly independent current injections is at most  $L$ . The selection of these independent patterns is not unique; any set of  $L$  linearly independent current injections can be used to represent all others through linear combinations. Nevertheless, in this work, we adopt the  $L$  SEMs as they offer a straightforward and intuitive choice for constructing arbitrary current injections.

### C. Conventional Implementation of GN Method

The inverse problem in both absolute EIT and time-difference EIT can be formulated within a unified optimization framework as:

$$\hat{\gamma} = \arg \min_{\gamma} \|\Delta \mathbf{v}(\gamma) - \Delta \mathbf{v}_{\text{obs}}\|^2, \quad (7)$$

where  $\Delta \mathbf{v}_{\text{obs}} = \mathbf{v}_{\text{obs}}^{t_2} - \mathbf{v}_{\text{obs}}^{t_1}$  denotes the observed voltage difference between two time points  $t_1$  and  $t_2$ , and  $\Delta \mathbf{v}(\gamma) = \mathbf{v}(\gamma) - \mathbf{v}(\gamma_{\text{ref}})$  represents the difference between the voltages simulated based on the conductivity model  $\gamma$  and the reference conductivity model  $\gamma_{\text{ref}}$ . The reference conductivity model  $\gamma_{\text{ref}}$  is normally chosen as a homogeneous conductivity distribution. By setting  $\mathbf{v}_{\text{obs}}^{t_1} = \mathbf{v}(\gamma_{\text{ref}})$ , (7) degrades into an absolute EIT problem.

The optimization problem in (7) is nonlinear and typically solved using iterative methods. A general update rule at the  $k$ -th iteration is given by:

$$\gamma_{k+1} = \gamma_k - \Delta \gamma, \quad (8)$$

where  $\Delta \gamma$  denotes the update to the conductivity distribution, the initial conductivity model  $\gamma_0$  can be set identical to  $\gamma_{\text{ref}}$ .

The update of the conductivity model  $\Delta \gamma$  can be computed using the Gauss-Newton (GN) method or other gradient-based optimization algorithms such as CG, L-BFGS, and ADAM. Among these methods, the GN algorithm is particularly favored in small-to-medium scale EIT problems due to its second-order convergence, often leading to more accurate reconstructions and faster convergence than the gradient-based methods. In the GN framework,  $\Delta \gamma$  is obtained by solving:

$$\mathbf{J}^* \mathbf{J} \Delta \gamma = \mathbf{J}^* (\Delta \mathbf{v}(\gamma_k) - \Delta \mathbf{v}_{\text{obs}}), \quad (9)$$

where  $\mathbf{J} = \frac{\partial \mathbf{v}(\gamma)}{\partial \gamma}|_{\gamma=\gamma_k}$  is known as the Jacobian matrix, typically computed using the adjoint method [32].

Each row of  $\mathbf{J}$  corresponds to a voltage measurement. The  $s$ -th row of  $\mathbf{J}$  can be numerically computed as:

$$\mathbf{J}_{s,:} = -\mathbf{e}_{\text{forward}} \odot \mathbf{e}_{\text{adjoint}} \odot \mathcal{S}, \quad (10)$$

where  $\odot$  denotes element-wise multiplication,  $\mathcal{S} \in \mathbb{R}^M$  represents the volume (or area in 2D) of each discretized element, and  $\mathbf{e}_{\text{forward}}$  and  $\mathbf{e}_{\text{adjoint}}$  are the electric fields derived from the gradients of the forward and adjoint potential fields:

$$\mathbf{e}_{\text{forward}} = \nabla \mathbf{u}_{\text{forward}}, \quad \mathbf{e}_{\text{adjoint}} = \nabla \mathbf{u}_{\text{adjoint}}. \quad (11)$$

As the forward problem of EIT is complex-self-adjoint, its adjoint problem shares the same numerical expression as (1) with a different “current injection”. Specifically, for the  $s$ -th voltage measurement, if the current is injected through electrodes  $a$  and  $b$ , and the voltage is measured between electrodes  $c$  and  $d$ , then:

- $\mathbf{u}_{\text{forward}}$  is the potential field generated by injecting a unit current from electrode  $a$  to electrode  $b$ .
- $\mathbf{u}_{\text{adjoint}}$  is the potential field generated by injecting a unit current from electrode  $c$  to electrode  $d$ .

In conventional GN implementations, both the Jacobian matrix  $\mathbf{J}$  and the approximate Hessian matrix  $\mathbf{J}^* \mathbf{J}$  must be explicitly computed and stored. Since each row of  $\mathbf{J}$  requires solving a pair of forward and adjoint problems in (4), followed by the element-wise operations in (11) and (10), the process becomes computationally and memory intensive—particularly in large-scale EIT systems with numerous measurements and highly discretized domains.

#### D. Matrix-Free GN Method

Instead of solving the normal (9) associated with the GN method to obtain  $\Delta\gamma$ , which imposes a significant memory and computational burden in large-scale EIT applications, alternative approaches [34], [35] propose solving the linearized system directly via matrix-free methods:

$$\mathbf{J}\Delta\gamma = \Delta\mathbf{v}(\gamma_k) - \Delta\mathbf{v}_{\text{obs}}. \quad (12)$$

In this work, we specifically propose the use of the Least Squares Minimal Residual (LSMR) method to solve (12). LSMR is an iterative Krylov subspace method [36], [37] specifically designed for large, sparse, or ill-conditioned systems. Since solving (12) via LSMR is analytically equivalent to solving the normal equations via the Minimal Residual (MINRES) method [38], we refer to the proposed approach as the Matrix-Free Gauss-Newton (MF-GN) method. Extensive analyses of the numerical stability and convergence properties of the LSMR method can be found in [39], [40].

Like other matrix-free solvers, LSMR necessitates the evaluation of the matrix-vector products  $\mathbf{J}\mathbf{v}$  and  $\mathbf{J}^*\mathbf{u}$  in each iteration, where  $\mathbf{v}$  and  $\mathbf{u}$  represent intermediate vectors. Given that convergence may require hundreds or even thousands of iterations, the primary computational challenge in the MF-GN framework lies in the efficient evaluation of  $\mathbf{J}$  or its adjoint. Leveraging the flexibility of our proposed SEM basis, we present the following efficient formulations for computing  $\mathbf{J}$  either row-wise (for  $\mathbf{J}\mathbf{v}$ ) or column-wise (for  $\mathbf{J}^*\mathbf{u}$ ):

$$\mathbf{J}_{s,:} = -\mathbf{P}_{s,:}\mathbf{E} \odot (\mathbf{Q}_{s,:}\mathbf{E}), \quad (13)$$

or

$$\mathbf{J}_{:,m} = -\mathbf{P}\mathbf{E}_{:,m} \odot (\mathbf{Q}\mathbf{E}_{:,m}). \quad (14)$$

Here,  $\mathbf{E} \in \mathbb{R}^{L \times M}$  is an electric field matrix, with its  $l$ -th row defined as:

$$\mathbf{E}_{l,:} = \mathbf{e}_l \odot S^{1/2}, \quad (15)$$

where  $\mathbf{e}_l$  is the electric field of the SEM problem activated by the  $l$ -th electrode, and  $S^{1/2}$  is the element-wise square root of the element volume (or area)  $S$ . Equations (13) and (14) are tailored for parallel computation of matrix-vector products  $\mathbf{J}\mathbf{v}$  and  $\mathbf{J}^*\mathbf{u}$ , respectively.

Matrices  $\mathbf{P}$  and  $\mathbf{Q}$  have dimensions  $S \times L$ , encoding the current injection pattern and voltage measurement patterns, respectively. For the  $s$ -th voltage measurement, assuming the current is injected from electrode  $a$  and exits from electrode  $b$ , and the voltage difference is measured between electrodes  $c$  and  $d$ , the  $s$ -th row of  $\mathbf{P}$  and  $\mathbf{Q}$  contains only two non-zero elements:

$$\mathbf{P}_{s,a} = 1, \mathbf{P}_{s,b} = -1, \mathbf{Q}_{s,c} = 1, \mathbf{Q}_{s,d} = -1. \quad (16)$$

Once the matrix  $\mathbf{E}$  is precomputed, evaluating  $\mathbf{J}$  via (13) or (14) becomes substantially more efficient than the traditional approach involving explicit forward and adjoint solves as described in (4), (11), and (10). This is primarily due to the sparsity of  $\mathbf{P}$  and  $\mathbf{Q}$ . However, it is important to note that in scenarios where multiple electrodes are used for current injection,  $\mathbf{P}$  becomes denser, leading to increased computational and memory

demands for evaluating the matrix-vector products required by LSMR.

#### E. Memory Cost and Computational Complexity

The following analysis of memory cost and computational complexity is conducted under the following assumptions: a) The imaging domain is discretized into  $M$  finite elements and  $N$  vertices. In most finite element meshes,  $N < M$ ; b) The system comprises  $L$  electrodes, leading to  $S = O(L^2)$  independent voltage measurements [41]; c) The set of  $S$  independent voltage measurements consists of  $S_c$  current injection patterns and  $S_v$  distinct voltage measurement patterns; d) The current is injected through  $p$  electrodes for each current injection pattern.

For the  $L$  electrodes, the maximum number of independent current injection pattern is  $L$ , resulting  $S_c \leq L$ . If each current injection pattern shares the same measurement pattern (such as the adjacent measurement [42]), then  $S_v \leq L$ . Thus, we have:

$$S_c + S_v \leq 2L \ll S = O(L^2). \quad (17)$$

However, for some specialized measurement protocols, such as the one generated by maximizing the “sensitivity volume” to get better imaging reconstructions [43], [44], each independent current injection pattern may have different voltage measurement patterns, which results:

$$S_c + S_v \leq S = O(L^2). \quad (18)$$

To maintain well-posedness in the inverse problem, the number of discretized elements  $M$  is generally kept smaller than the number of measurements  $S$ . However, with proper regularization techniques, configurations where  $M > S$  can also yield stable solutions. This leads to the fundamental parameter relationship:

$$L \ll N < M \lesssim S = O(L^2) \quad (19)$$

where the  $M \lesssim S$  notation indicates that either relationship may hold.

1) *Memory Requirements:* In the conventional GN framework, the dominant memory consumption arises from storing the Jacobian matrix  $\mathbf{J} \in \mathbb{R}^{S \times M}$  and its associated Hessian approximation  $\mathbf{J}^*\mathbf{J} \in \mathbb{R}^{M \times M}$ . These require memory on the order of  $O(SM)$  and  $O(M^2)$ , respectively. The total memory cost for the conventional GN method  $\phi_{\text{GN}}$  is thus:

$$\phi_{\text{GN}} = O(SM + M^2). \quad (20)$$

In contrast, the proposed MF-GN method significantly reduces memory usage by avoiding the explicit storage of the Jacobian matrix. Instead, the main memory components include the precomputed electric fields  $\mathbf{E} \in \mathbb{R}^{L \times M}$  for the  $L$  SEM problems, requiring  $O(LM)$  memory; the current injection pattern matrix  $\mathbf{P} \in \mathbb{R}^{S \times p}$ , requiring  $O(pS)$  memory; and the voltage measurement configuration matrix  $\mathbf{Q} \in \mathbb{R}^{S \times 2}$ , requiring  $O(2S)$  memory. Accordingly, the total memory cost of the matrix-free method  $\phi_{\text{MF-GN}}$  is:

$$\phi_{\text{MF-GN}} = O(LM + (p+2)S). \quad (21)$$

In typical EIT applications, current is injected through pairs of electrodes ( $p = 2$ ).

2) *Computation Complexity*: In the conventional GN method, the Jacobian matrix  $\mathbf{J}$  must be first constructed, which requires to perform LU decomposition of the sparse system matrix  $\mathbf{A}$  for once, and solving (4) for  $(S_c + S_v)$  times to obtain  $\mathbf{u}_{\text{forward}}$  and  $\mathbf{u}_{\text{adjoint}}$  [33], [35], and finally assembling  $\mathbf{J}$  row-by-row following (10) and (11). For 3D FEM problems of EIT that build on tetrahedron mesh, the LU decomposition of  $\mathbf{A}$  requires  $O((N + L)^2)$  Floating Point Operations (FLOPs), and solving (4) per-time requires  $O((N + L)^{4/3})$  FLOPs [45], [46]. Assembling  $\mathbf{J}$  row-by-row requires  $O(SM)$  FLOPs. Consequently, the overall computational complexity  $\psi_{\mathbf{J}}$  of constructing the Jacobian matrix is:

$$\psi_{\mathbf{J}} = O(N^2) + O((S_c + S_v)(N + L)^{4/3}) + O(SM). \quad (22)$$

After building the Jacobian matrix, the major computation cost of conventional GN method arises from constructing the approximate Hessian matrix  $\mathbf{J}^* \mathbf{J}$ , which requires  $O(SM^2)$  FLOPs. Finally, solving the linear system in (9) using a direct solver adds  $O(M^3)$  FLOPs. Therefore, the overall computational complexity of the conventional GN method, denoted by  $\psi_{\text{GN}}$ , is given by:

$$\psi_{\text{GN}} = \psi_{\mathbf{J}} + O(SM^2) + O(M^3). \quad (23)$$

When employing an iterative solver for (9),  $\psi_{\text{GN}}$  becomes:

$$\psi_{\text{GN}} = \psi_{\mathbf{J}} + O(SM^2) + O(fM^2), \quad (24)$$

where  $f$  denotes the iteration count required by the iterative solver for convergence. For the EIT applications that satisfy (17),  $\psi_{\mathbf{J}}$  is negligible in (23) and (24).

In the proposed MF-GN method, we also need to perform the LU decomposition of  $\mathbf{A}$  and then precompute the electric field matrix  $\mathbf{E}$  in (15) by solving (4) for each of the  $L$  SEM problem, requiring  $O((L(N + L)^{4/3}))$  FLOPs. Then, the matrix-vector products  $\mathbf{J}\mathbf{w}$  and  $\mathbf{J}^*\hat{\mathbf{w}}$  required by the LSMR algorithm are computed on-the-fly using (13) or (14), with a per-iteration cost of  $O(2(p + 3)SM)$  FLOPs. Assuming LSMR requires  $f$  iterations to converge, the overall complexity of the MF-GN method  $\psi_{\text{MF-GN}}$  is:

$$\psi_{\text{MF-GN}} = O(N^2) + O(L(N + L)^{4/3}) + O(2(p + 3)fSM). \quad (25)$$

The iteration count  $f$  in (24) and (25) is influenced by the condition numbers of  $\mathbf{J}^* \mathbf{J}$  and  $\mathbf{J}$ , respectively. Although both matrices are generally ill-conditioned, the use of regularization and preconditioning techniques, discussed in the following subsection, can substantially improve their conditioning. As a result, in large-scale EIT applications, the iteration count  $f$  often satisfies:

$$f \ll \min(S, M). \quad (26)$$

Under the condition described in (26), the computational complexity of the MF-GN method given in (25) can be lower than that of the conventional GN method described in (23) and (24). This highlights the superior computational efficiency of the proposed MF-GN approach in large-scale EIT scenarios.

## F. Regularization and Precondition

Due to the ill-posed nature of the EIT inverse problem (7), both the Jacobian matrix  $\mathbf{J}$  and its normal form  $\mathbf{J}^* \mathbf{J}$  are typically ill-conditioned. To address this issue, the problem is commonly regularized as follows:

$$\hat{\gamma} = \arg \min_{\gamma} \|\Delta \mathbf{v}(\gamma) - \Delta \mathbf{v}_{\text{obs}}\|^2 + \lambda \Psi(\gamma), \quad (27)$$

where  $\Psi(\gamma)$  is a regularization functional that encodes prior information, and  $\lambda > 0$  is the regularization parameter that balances data fidelity and regularization.

Incorporating this term modifies the GN update (9) into the following regularized form:

$$(\mathbf{J}^* \mathbf{J} + \lambda \mathbf{R}) \Delta \gamma = \mathbf{J}^* (\Delta \mathbf{v}(\gamma_k) - \Delta \mathbf{v}_{\text{obs}}), \quad (28)$$

where  $\mathbf{R} = \frac{\partial^2 \Psi(\gamma_k)}{\partial \gamma_k^2}$  denotes the Hessian (or an approximation thereof) of the regularization functional at the current iteration.

For some commonly used regularization strategies in EIT, such as Tikhonov regularization and Total Variation (TV) regularization [47], [48], the corresponding  $\mathbf{R}$  is typically sparse and can be (approximately) decomposed as  $\mathbf{R} = \mathcal{R}^* \mathcal{R}$ . Thus (28) can be rewritten in the following form for implementing the matrix-free LSMR method:

$$\begin{bmatrix} \mathbf{J} \\ \sqrt{\lambda} \mathcal{R} \end{bmatrix} \Delta \gamma = \begin{bmatrix} \Delta \mathbf{v}(\gamma_k) - \Delta \mathbf{v}_{\text{obs}} \\ \mathbf{0} \end{bmatrix}. \quad (29)$$

The regularization term in (29) can alternatively be formulated to directly constrain  $\Delta \gamma$  rather than  $\gamma$  via  $\Psi(\gamma)$ . However, such regularization approaches require careful consideration, as their influence on  $\gamma$  may lack interpretability. Moreover, this modification disrupts the theoretical equivalence between (28) and (29).

To improve the convergence behavior of iterative solvers like LSMR, we solve a right-preconditioned version of (29):

$$\begin{bmatrix} \mathbf{J} \\ \sqrt{\lambda} \mathcal{R} \end{bmatrix} \mathcal{P} \mathbf{y} = \begin{bmatrix} \Delta \mathbf{v}(\gamma_k) - \Delta \mathbf{v}_{\text{obs}} \\ \mathbf{0} \end{bmatrix}, \quad (30)$$

where the solution is recovered via:

$$\Delta \gamma = \mathcal{P} \mathbf{y}, \quad (31)$$

and  $\mathcal{P}$  is the right preconditioner.

For simplicity and computational tractability, we propose using a diagonal preconditioner defined by the reversed column-wise  $\ell_2$  norms of the augmented Jacobian matrix:

$$\mathcal{P} = \text{diag} \left( \left\| \begin{bmatrix} \mathbf{J} \\ \sqrt{\lambda} \mathcal{R} \end{bmatrix}_{:,m} \right\|^{-1} \right)_{m=1}^M, \quad (32)$$

where  $\|\cdot\|$  denotes the  $\ell_2$  norm.

As long as  $\mathbf{R}$ ,  $\mathcal{R}$ , and  $\mathcal{P}$  are implemented as sparse matrices, the memory and computational cost analysis presented earlier remains valid, even with regularization and preconditioning applied. The pseudocode for solving the preconditioned system (30) using a matrix-free LSMR algorithm is provided in Algorithm 1.

---

**Algorithm 1:** Computation of  $\Delta\gamma$  at the  $k$ -th MF-GN Iteration for Time-Difference EIT.

---

**E** SEM electric field matrix defined by (15)  
**P**, **Q** current injection and voltage measurement pattern matrices defined by (16)  
**R** regularization operator  
 $\lambda$  regularization weight  
**Require:**  $\Delta\mathbf{v}_{\text{obs}}$  measured voltage difference between time point  $t_1$  and  $t_2$   
 $\Delta\mathbf{v}(\gamma_k)$  simulated voltage difference between  $\gamma_k$  and reference conductivity model  $\gamma_{\text{ref}}$   
 $\text{tol}$  LSMR stopping tolerance  
 $f_{\text{max}}$  maximum LSMR iterations  
**Ensure:**  $\Delta\gamma$   
1: **Compute the preconditioning matrix**  $\mathcal{P}$ :  
2: **for**  $m = 1$  to  $M$  **do**  
3:  $\mathbf{j} \leftarrow -(\mathbf{P}\mathbf{E}_{:,m}) \odot (\mathbf{Q}\mathbf{E}_{:,m})$   
4:  $\mathcal{P}_{m,m} \leftarrow 1/\sqrt{\|\mathbf{j}\|_2^2 + \lambda\|\mathbf{R}_{:,m}\|_2^2}$   
5: **end for**  
6: **Define matrix-free matrix-vector product operators:**  
7: **function** ForwardProduct  $\mathbf{v}$   
8:  $\mathbf{u} \leftarrow \mathbf{0}$   $\triangleright \mathbf{u}$  is a vector in length of  $S + M$   
9: **for**  $s = 1$  to  $S$  **do**  
10:  $\mathbf{u}_s \leftarrow -(\mathcal{P}_{m,m})(\mathbf{P}_{s,:}\mathbf{E} \odot \mathbf{Q}_{s,:}\mathbf{E}) \cdot \mathbf{v}$   
 $\triangleright \mathbf{J}$  is implicitly built row-wise using (13)  
11: **end for**  
12:  $\mathbf{u}_{S+1:S+M} \leftarrow \sqrt{\lambda}\mathbf{R}(\mathcal{P}\mathbf{v})$   
14: **return**  $\mathbf{u}$   
15: **end Function**  
16: **function** AdjointProduct  $\mathbf{u}$   
17:  $\mathbf{v} \leftarrow \mathbf{0}$   $\triangleright \mathbf{v}$  is a vector in length of  $M$   
18: **for**  $m = 1$  to  $M$  **do**  
19:  $\mathbf{v}_m \leftarrow -\text{Conj}(\mathbf{P}\mathbf{E}_{:,m} \odot \mathbf{Q}\mathbf{E}_{:,m}) \cdot \mathbf{u}_{1:S}$   
 $\triangleright \mathbf{J}$  is implicitly built column-wise using (14)  
20: **end for**  
21:  $\mathbf{v} \leftarrow \mathcal{P}(\mathbf{v} + \sqrt{\lambda}\mathbf{R}^*\mathbf{u}_{S+1:S+M})$   
23: **return**  $\mathbf{v}$   
24: **end Function**  
25: **Solve:**  
26:  $\mathbf{b} \leftarrow \begin{bmatrix} \Delta\mathbf{v}(\gamma_k) - \Delta\mathbf{v}_{\text{obs}} \\ \mathbf{0} \end{bmatrix}$   
27:  $\mathbf{y} \leftarrow \text{LSMR}(\text{ForwardProduct}, \text{AdjointProduct}, \mathbf{b}, \text{tol}, f_{\text{max}})$   
28: **return**  $\Delta\gamma \leftarrow \mathcal{P}\mathbf{y}$

---

In our work, the two matrix-vector product operators in Algorithm 1 FORWARDPRODUCT and ADJOINTPRODUCT are implemented in parallelization using OpenMP to ensure efficient loop execution. These two operators can also be implemented using GPU parallel computing for further performance improvement. MATLAB and Python implementations are discouraged for these computationally intensive components due to their inherent performance limitations in loop operations. Some scientific computing libraries, such as SciPy in Python and PETSc in C++, support matrix-free implementations of linear system solvers, including the LSMR method, where the solver operates

on user-defined matrix-vector product operators instead of an explicitly formed system matrix.

### III. NUMERICAL EXPERIMENTS

#### A. Comparison of Reconstruction Accuracy

This section presents numerical experiments to evaluate the reconstruction accuracy of the proposed MF-GN method in comparison with conventional GN, L-BFGS, and ADAM methods. Considering neural NN-regularized methods for EIT are not yet standardized and their performance depends strongly on architectural choices, we provide a brief discussion and a representative comparison in Appendix A for completeness.

The numerical experiment simulates organoid imaging using EIT with micro-electrodes [13]. A synthetic organoid model is placed within a cubic chamber filled with phosphate-buffered saline (PBS), a bio-compatible medium that can be used in EIT for imaging bio-objects. The organoid model consists of an outer shell and two inner cores. The complex-valued conductivities are set to  $2.0 + 1.0i$  S/m for the PBS solution,  $1.0 + 0.6i$  S/m for the shell, and  $0.5 + 0.2i$  S/m for the two cores.

A total of 400 micro-electrodes, arranged in a  $10 \times 10$  grid on each of the four side walls of the cubic chamber, are used for data acquisition. Voltage measurements are acquired following the Sheffield protocol [42] yielding 79,400 independent measurements for conductivity reconstruction. The experimental configuration and the geometry of the organoid model are illustrated in the first column of Fig. 1.

Unlike conventional EIT applications that focus solely on distinguishing target objects from background media (e.g., detecting 3D tissue models in culture medium [49] or lung imaging in the human chest [50]), this task is more demanding. The objective is not only to localize the organoid within the PBS but also to resolve its internal structures, necessitating high reconstruction accuracy.

For the forward simulation, the ground-truth model is discretized into approximately 170,000 tetrahedral elements, and the point electrode model (PEM) is solved using the finite element method (FEM) to generate synthetic voltage data. In the inversion stage, an initial homogeneous model (with PBS conductivity) discretized into around 72,000 tetrahedrons is used. The four reconstruction methods (MF-GN, GN, L-BFGS, ADAM), with regularization function  $\Psi(\gamma) = \|\nabla\gamma\|^2$  and regularization weight  $\lambda = 10^{-2}$ , are then applied to recover the conductivity distribution. The GN method is implemented by solving (28) via LU decomposition, as in all other experiments in this work.

The reconstruction accuracy is quantitatively assessed by projecting the recovered conductivity distributions onto the ground-truth mesh and computing the Structural Similarity Index (SSIM) and Peak Signal-to-Noise Ratio (PSNR).

Fig. 2 compares the convergence behavior and reconstruction accuracy of the four methods (MF-GN, GN, L-BFGS, ADAM) by plotting the cost function, PSNR, and SSIM in each iteration. The MF-GN and GN methods were executed for 5 iterations, while L-BFGS and ADAM were run for 1000 iterations.

As shown in Fig. 2, the cost function, SSIM, and PSNR curves for MF-GN and GN are nearly identical, suggesting

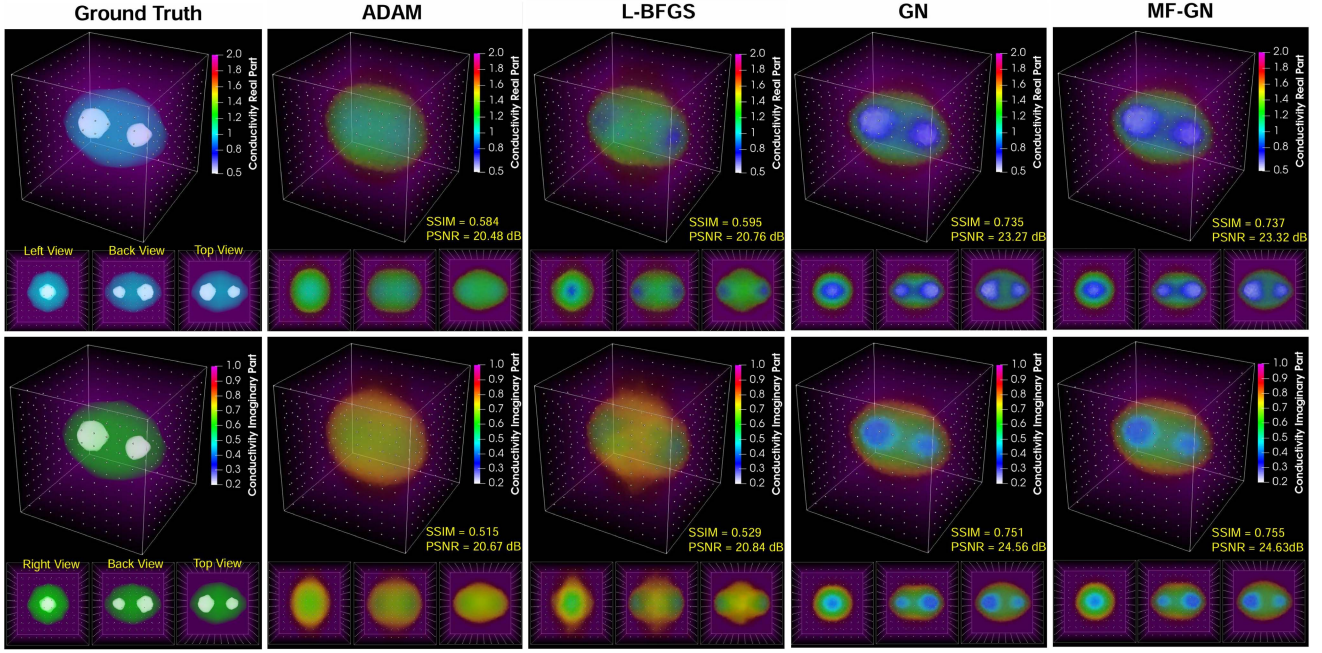


Fig. 1. Comparison of various reconstruction methods. The first and second rows correspond to the real and imaginary parts of the complex-valued conductivity, respectively. From left to right, the first column presents the ground truth model, while the second and third columns show the ADAM and L-BFGS reconstruction after 1000 iterations, respectively. The fourth and fifth column display the reconstruction results obtained using GN and MF-GN after 5 iterations, respectively.

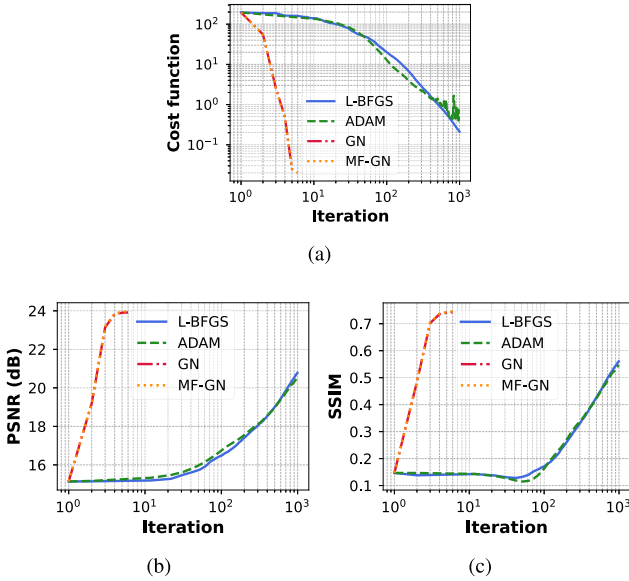


Fig. 2. Performance metrics as a function of iteration: (a) Cost function, (b) PSNR, and (c) SSIM. The performance metrics of the first iteration are calculated based on the initial model. PSNR and SSIM values represent averages computed separately for the reconstructed real and imaginary components of the complex-valued conductivity.

similar performance in both reconstruction efficiency and accuracy. In contrast, L-BFGS and ADAM exhibit significantly slower convergence speeds than GN and MF-GN. Based on the cost function convergence trend in Fig. 2(a), L-BFGS and ADAM may require up to 10,000 iterations to reach the same convergence level as GN and MF-GN, which is computationally impractical for this numerical experiment. Due to incomplete convergence after 1,000 iterations, the final SSIM and PSNR

values for L-BFGS and ADAM remain notably lower than those of MF-GN and GN, reflecting reduced reconstruction accuracy.

Fig. 1 displays the final conductivity distributions recovered by each method. The results confirm that MF-GN and GN produce highly similar reconstructions, both accurately resolving the organoid's shell and internal structures. Conversely, L-BFGS achieves only partial recovery of the internal cores, while ADAM fails to reconstruct them effectively. These observations align with the quantitative metrics presented in Fig. 2, further validating the superior performance of MF-GN and GN in this imaging task.

### B. Comparison of Computational Cost

We conducted a series of six numerical experiments with progressively increasing scale to evaluate the inversion performance using the same synthetic organoid model introduced Section III-A. The experiments were systematically designed with increasing complexity, where the electrode configurations expanded from  $4 \times 5 \times 5$  in Experiment 1 to  $4 \times 10 \times 10$  in Experiment 6. This progression resulted in a corresponding increase in independent voltage measurements from 4,850 to 79,400 across the experimental series. To maintain appropriate resolution, the conductivity models used in the inversion were discretized using tetrahedral meshes ranging from 4,400 to 72,169 elements. Both the proposed MF-GN method and the conventional GN approach were executed for five iterations on a Linux cluster comprising 1,024 GB of RAM and 128 CPU cores, with each core having a frequency of 2.35 GHz. The comprehensive evaluation metrics, including computational costs (memory usage and processing time) and reconstruction quality indicators (PSNR and SSIM), are presented in Table I.

TABLE I  
COMPUTATIONAL PERFORMANCE AND RECONSTRUCTION QUALITY COMPARISON BETWEEN GN AND MF-GN METHODS

| Idx | Elec. $L$       | Meas. $S$ | Vert. $N$ | Elem. $M$ | GN           |                |           |       | MF-GN      |                |           |       |                |
|-----|-----------------|-----------|-----------|-----------|--------------|----------------|-----------|-------|------------|----------------|-----------|-------|----------------|
|     |                 |           |           |           | Mem. (GB)    | Time (s)       | PSNR (dB) | SSIM  | Mem. (GB)  | Time (s)       | PSNR (dB) | SSIM  | LSMR Iter. $f$ |
| 1   | $4 \times 5^2$  | 4850      | 833       | 4400      | 2.1          | 22.2           | 17.48     | 0.474 | 0.8        | 50.6           | 17.84     | 0.463 | 183            |
| 2   | $4 \times 6^2$  | 10 152    | 2169      | 9652      | 6.2          | 138.2          | 22.40     | 0.660 | 0.9        | 298.2          | 22.46     | 0.658 | 198            |
| 3   | $4 \times 7^2$  | 18 914    | 3676      | 17 301    | 20.1         | 393.1          | 23.14     | 0.699 | 1.2        | 720.4          | 23.16     | 0.697 | 202            |
| 4   | $4 \times 8^2$  | 32 384    | 5938      | 28 648    | 55.8         | 1044.0         | 23.46     | 0.715 | 1.6        | 1743.5         | 23.48     | 0.714 | 195            |
| 5   | $4 \times 9^2$  | 52 002    | 9158      | 45 901    | 134.9        | 5012.8         | 23.74     | 0.730 | 2.5        | 5728.1         | 23.75     | 0.731 | 197            |
| 6   | $4 \times 10^2$ | 79 400    | 14 143    | 72 169    | <b>330.9</b> | <b>19953.9</b> | 23.91     | 0.743 | <b>4.2</b> | <b>12412.2</b> | 23.98     | 0.746 | 204            |

Note: All experiments were conducted on a computing cluster equipped with 1,024 GB of RAM and four AMD EPYC 7452 processors, providing a total of 128 CPU cores.  $f$ : Average LSMR iterations among the five MF-GN iterations. SSIM and PSNR are the averages of the real and imaginary parts of the complex-valued conductivity.

The bold values emphasizes that, for the large scale EIT problem, MF-GN achieves dramatically lower memory consumption with competitive or superior runtime, which is the central claim of the paper.

Based on the SSIM and PSNR presented in Table I, the MF-GN method achieves reconstruction accuracy comparable to that of the conventional GN approach, as inferred from consistent LSMR iteration counts across all experiments. Regarding computational efficiency, GN outperforms MF-GN in Experiments 1 to 5, where the problem size is relatively small. However, MF-GN demonstrates superior scalability and becomes significantly faster in Experiments 6, which involve larger problem instances. Furthermore, MF-GN exhibits a substantial advantage in memory efficiency across all experiments, with memory usage being notably lower than that of GN, particularly in large-scale scenarios. This confirms MF-GN's suitability for large-scale EIT applications.

### C. Comparison of Robustness

Based on Experiment 4, which employs  $4 \times 8 \times 8$  electrodes as described in Section III-B, we evaluate the robustness of the GN and MF-GN methods with respect to voltage measurement noise, electrode location errors, and mesh resolution. Both methods solve the same optimization problem (27) with regularization function  $\Psi(\gamma) = \|\nabla \gamma\|^2$  and weight  $\lambda = 10^{-2}$ .

To assess robustness to voltage measurement noise, Gaussian noise is added to the simulated voltage data generated from the ground-truth model. The signal-to-noise ratio (SNR) ranges from 25 dB to 80 dB. The resulting PSNR and SSIM are shown in Fig. 3(a) and (b), where MF-GN achieves slightly higher PSNR and SSIM than the conventional GN method. Robustness to electrode location errors is evaluated by introducing random perturbations to the electrode coordinates used in the inversion. The relative position error varies from 0% to 2% of the side length of the cubic imaging domain. The corresponding PSNR and SSIM are presented in Fig. 3(c) and (d), indicating that MF-GN yields marginally higher PSNR but slightly lower SSIM. To evaluate robustness to mesh resolution, the mesh sizes was varied from 5% to 7% of the imaging-domain side length. The PSNR and SSIM as a function of mesh resolution are shown in Fig. 3(e) and (f), exhibiting a similar trend, with MF-GN producing slightly higher PSNR but lower SSIM.

Overall, the differences in PSNR and SSIM between GN and MF-GN are negligible. Fig. 4 shows reconstructions of the real part of the complex-valued conductivity under various

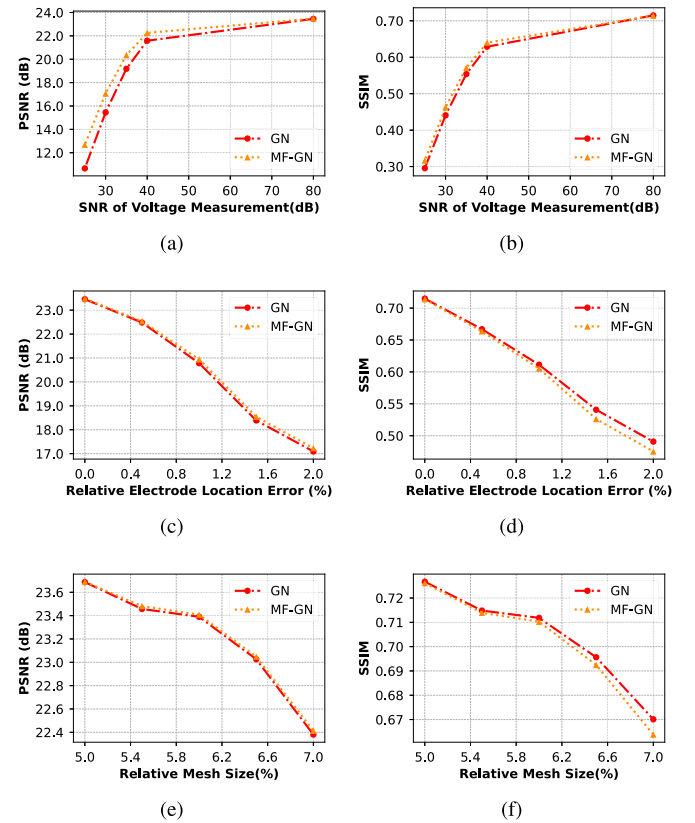


Fig. 3. PSNR and SSIM of reconstructions under: (a)-(b) varying voltage measurement SNR, (c)-(d) varying electrode location errors, and (e)-(f) varying mesh resolutions.

error sources, where the results of GN and MF-GN are visually indistinguishable. This is expected, as discussed in Section II-D, since the MF-GN method is mathematically equivalent to the GN method.

## IV. REAL EXPERIMENT

To evaluate the practical applicability of the proposed MF-GN method, we conducted two real-world experiments.

The first experiment mirrors the numerical organoid tomography experiment described in Section III-A. The measurement setup consists of a  $1\text{cm} \times 1\text{cm} \times 1\text{cm}$  cubic chamber, with 32

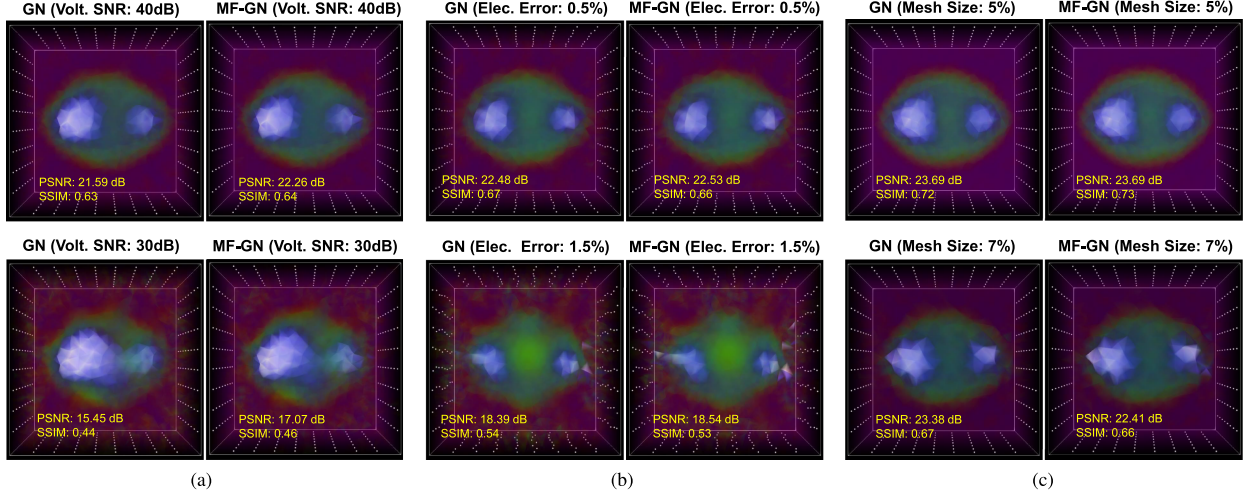


Fig. 4. Reconstructions of GN and MF-GN under: (a) varying voltage measurement SNR, (b) varying electrode location errors, and (c) varying mesh resolutions. The ground truth corresponds to the top-view of the real part of the complex-valued conductivity presented in Fig. 1.

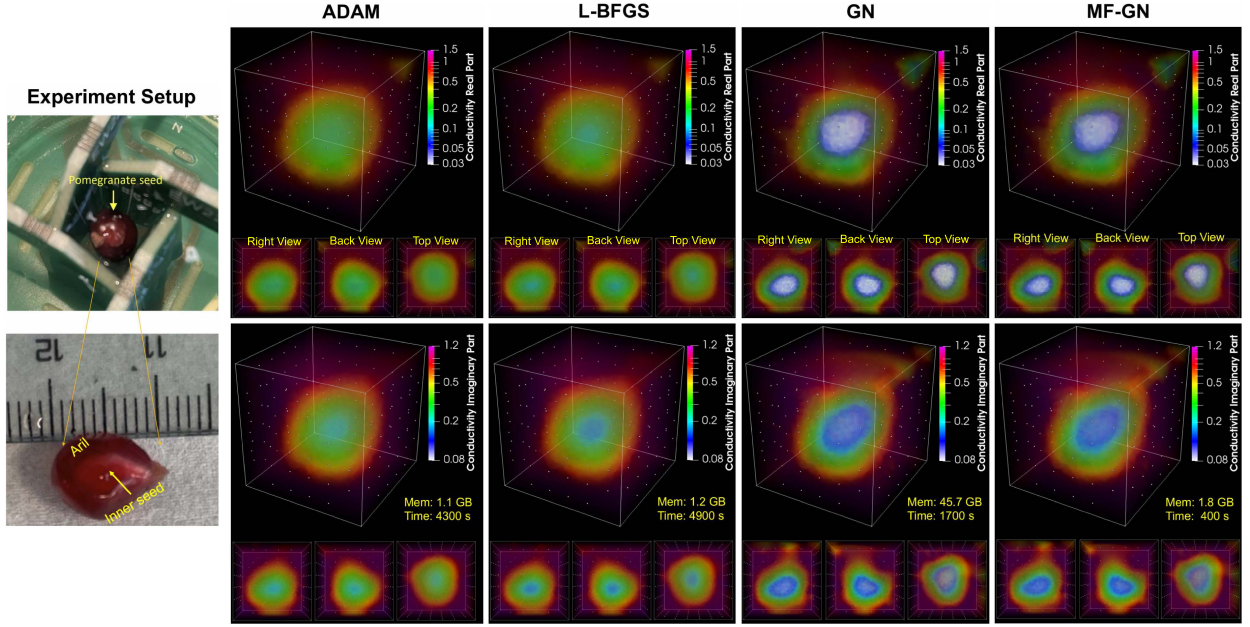


Fig. 5. Reconstruction results for a dual-layer pomegranate (real application A). The first column shows the experimental setup. The second and third columns present reconstructions from ADAM and L-BFGS after 1000 iterations, while the fourth and fifth columns show GN and MF-GN results after 5 iterations. All methods employ the same regularization function  $\Psi(\gamma) = \|\nabla\gamma\|^2$  with  $\lambda = 10^{-2}$ .

micro-electrodes mounted on each of four side walls, totaling 128 electrodes. The imaging target is a pomegranate seed exhibiting a dual-layer structure, comprising an outer aril and an inner seed (see Fig. 5).

The chamber was initially filled with PBS, and 8,000 independent voltage measurements were collected using the Sheffield protocol under an injected alternating current at 50 kHz. Subsequently, the pomegranate was placed into the chamber, and the measurement procedure was repeated. The imaging domain is discretized into around 35,000 tetrahedron elements. Time-difference EIT was then performed using the MF-GN, GN, L-BFGS, and ADAM methods. The initial conductivity model  $\gamma_0$  and reference conductivity model  $\gamma_{ref}$  are assumed to be

homogeneous and identical to each other, with the conductivity estimated as:

$$\gamma_{ref} = \gamma_0 = \frac{2(p + iq)}{p^2 + q^2}, \quad (33)$$

where

$$p = \frac{\langle |\Re(\mathbf{v}_{obs}^{t1})| \rangle}{\langle |\Re(\mathbf{v}_{baseline})| \rangle}, \quad q = \frac{\langle |\Im(\mathbf{v}_{obs}^{t1})| \rangle}{\langle |\Im(\mathbf{v}_{baseline})| \rangle}. \quad (34)$$

Here,  $\langle \cdot \rangle$  denotes the mean,  $|\cdot|$  the absolute value, and  $\Re(\cdot)$  and  $\Im(\cdot)$  the real and imaginary parts, respectively.  $\mathbf{v}_{obs}^{t1}$  is the measured voltage without targets, and  $\mathbf{v}_{baseline}$  is the simulated voltage for a homogeneous medium with conductivity 1.0 +

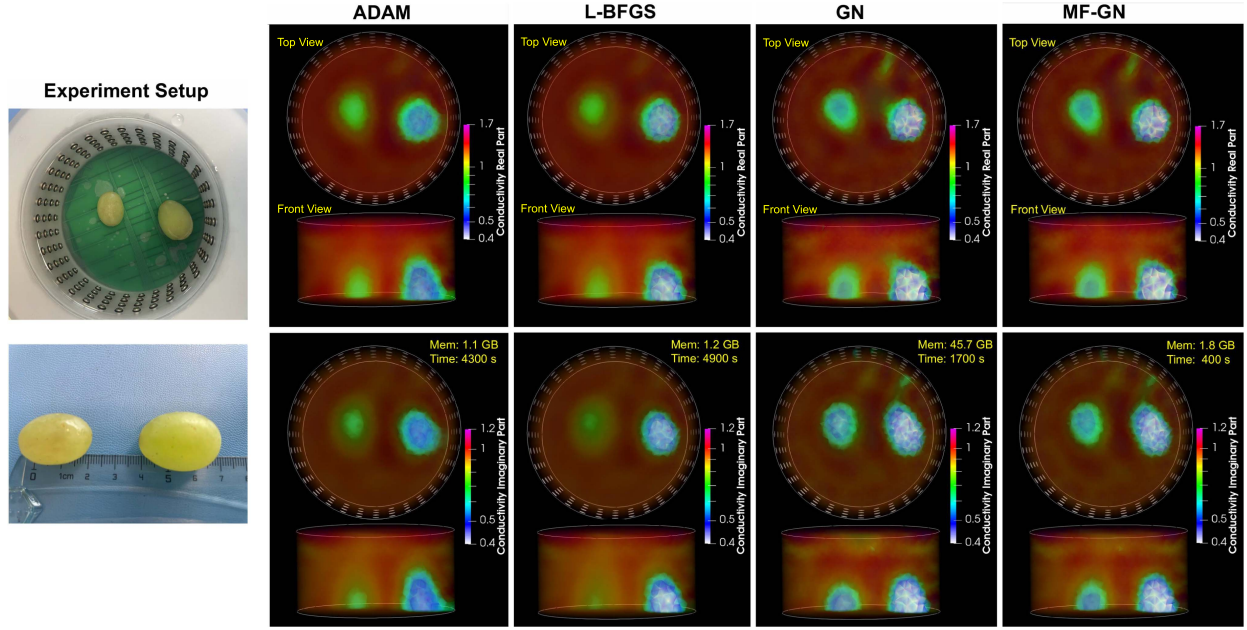


Fig. 6. Reconstruction results for two grapes (real application B). From left to right, columns show: experimental configuration, ADAM (1000 iterations), L-BFGS (1000 iterations), GN (5 iterations), and MF-GN (5 iterations), respectively. All methods employ the same regularization function  $\Psi(\gamma) = \|\nabla\gamma\|^2$  with  $\lambda = 10^{-3}$ .

1.0i S/m. The initial model obtained from (33) and (34) yields simulated voltages whose real and imaginary components have the same mean amplitude as the corresponding experimental voltages measured in the absence of objects.

The PEM is solved to get simulated voltage during the inversion as the size of the micro-electrodes is negligible compared to that of the computational domain. The reconstructions are shown in Fig. 5. As illustrated, both the MF-GN and GN methods yield highly similar reconstructions that clearly resolve the dual-layer structure of the pomegranate. In contrast, reconstructions obtained via L-BFGS and ADAM exhibit significantly reduced conductivity contrast between the aril and the inner seed, making the dual-layer structure more difficult to discern.

In terms of computational performance, both MF-GN and GN were executed for 5 iterations, with total runtime of about 400 s and 1,700 s, respectively. L-BFGS and ADAM were run for 1,000 iterations, resulting in runtime exceeding that of MF-GN by more than 10 times. The corresponding resident memory usage for MF-GN, GN, L-BFGS, and ADAM was 1.8 GB, 45.7 GB, 1.2 GB, and 1.1 GB, respectively.

In the second experiment, the measurement interface consists of a cylindrical tank with a diameter of 10 cm. A total of 128 electrodes are uniformly distributed along the side wall of the cylinder, arranged in four horizontal rings from bottom to top. Two grapes, one small and one large, were placed at the bottom of the tank to serve as imaging targets.

The measurement protocol, the current injection frequency, and the reconstruction procedures were identical to those used in the first experiment. The initial and reference conductivity are set according to (33). The CEM is adapted in the inversion considering the electrodes are relatively large. Photographs of the experimental setup, along with the corresponding reconstructions, are shown in Fig. 6.

Consistent with the results from the pomegranate experiment, the reconstructions produced by the MF-GN and GN methods closely resemble each other. Both methods successfully captured the shape, size, and orientation of the two grapes, outperforming the L-BFGS and ADAM methods, which yielded reconstructions with less contrast.

The memory usage and computation times for all four methods were comparable to those observed in the first experiment. This is expected, as the number of voltage measurements and the complexity of the inversion model (in terms of mesh resolution and element count) were similar in both experiments.

## V. CONCLUSION AND DISCUSSION

In this study, we tackle the computational and memory bottlenecks of large-scale EIT reconstruction by introducing a novel framework based on the SEM and MF-GN method. The SEM significantly reduces the computational complexity involved in Jacobian matrix calculations, which are typically a major performance bottleneck in the GN method. Building on this, the MF-GN method avoids the need for explicit formation and storage of large Jacobian matrices, leading to substantial reductions in both memory usage and computational cost. Despite these efficiencies, MF-GN maintains the high reconstruction accuracy characteristic of conventional Gauss-Newton methods. Extensive numerical simulations and real-world experiments validate the effectiveness of the proposed method, demonstrating its robustness, scalability, and practicality for solving large-scale EIT problems.

Although this work advances the reduction of computational cost in Newton-type optimization for large-scale EIT with high-density electrode arrays, the cost-effectiveness of resolution improvement via electrode densification is inherently limited

by the diffusive nature of the forward problem. Consequently, further gains in reconstruction quality can not rely on hardware densification alone. Future work will therefore focus on integrating the MF-GN framework with advanced regularization strategies, particularly deep learning-based priors that incorporate structural information from complementary imaging modalities (e.g., CT, MRI, or ultrasound), to mitigate the ill-posedness of the inverse problem and enhance reconstruction fidelity, edge preservation, and spatial resolution beyond hardware-imposed limits.

## APPENDIX A COMPARISON BETWEEN MF-GN AND NN-REGULARIZED METHODS

In recent years, NNs have been introduced into EIT as implicit regularizers. In NN-regularized EIT, the conductivity distribution is parameterized by a neural network as

$$\gamma = \mathcal{N}(\mathbf{h}, \boldsymbol{\theta}), \quad (35)$$

where  $\mathcal{N}(\mathbf{h}, \boldsymbol{\theta})$  denotes a neural network with input  $\mathbf{h}$  and parameters  $\boldsymbol{\theta}$ .

Substituting (35) into (27), the EIT inverse problem can be reformulated as an optimization problem with respect to  $\boldsymbol{\theta}$ :

$$\hat{\boldsymbol{\theta}} = \arg \min_{\boldsymbol{\theta}} \|\Delta \mathbf{v}(\mathcal{N}(\mathbf{h}, \boldsymbol{\theta})) - \Delta \mathbf{v}_{\text{obs}}\|^2 + \lambda \Psi(\mathcal{N}(\mathbf{h}, \boldsymbol{\theta})), \quad (36)$$

where the first term enforces data fidelity and the second term encodes prior information through the regularizer  $\Psi(\cdot)$ .

The above optimization problem (36) can be solved using gradient-based optimization methods. Then the reconstructed conductivity distribution is obtained as  $\hat{\gamma} = \mathcal{N}(\mathbf{h}, \hat{\boldsymbol{\theta}})$ .

Following the chain rule, the gradient of the loss function with respect to the network parameters  $\boldsymbol{\theta}$  is expressed as:

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{\theta}} = \frac{\partial \mathcal{L}}{\partial \gamma} \frac{\partial \gamma}{\partial \boldsymbol{\theta}} = \frac{\partial \mathcal{L}}{\partial \gamma} \frac{\partial \mathcal{N}(\mathbf{h}, \boldsymbol{\theta})}{\partial \boldsymbol{\theta}}, \quad (37)$$

where  $\mathcal{L}(\boldsymbol{\theta}) = \|\Delta \mathbf{v}(\mathcal{N}(\mathbf{h}, \boldsymbol{\theta})) - \Delta \mathbf{v}_{\text{obs}}\|^2 + \lambda \Psi(\mathcal{N}(\mathbf{h}, \boldsymbol{\theta}))$ .

To compute this gradient in practice, we first calculate  $\frac{\partial \mathcal{L}}{\partial \gamma}$  using the adjoint method, as in traditional gradient-based EIT. This intermediate gradient is then supplied as the initial gradient to reverse-mode of automatic differentiation (AD) frameworks (e.g., PyTorch) [51], [52], which propagate it through the NN to obtain the final gradient  $\frac{\partial \mathcal{L}}{\partial \boldsymbol{\theta}}$ . Thus, the main additional computational complexity compared to traditional gradient-based EIT arises from this reverse-mode AD on the NN.

In response to the increasing number of NN-regularized methods [53], [54], this paper provides a concise overview of three representative methods reported in the literature.

The first is the Deep Image Prior (DIP) method [21], [22], [55], in which  $\mathcal{N}$  is a convolutional neural network (CNN) and the input  $\mathbf{h}$  is random noise. Although DIP has been shown to improve reconstruction accuracy, it suffers from spectral bias, which limits its ability to recover high-frequency features. Furthermore, the CNN output is constrained to a rectangular or cubic image domain, and this necessitates interpolation between the CNN image and the finite element conductivity model. The

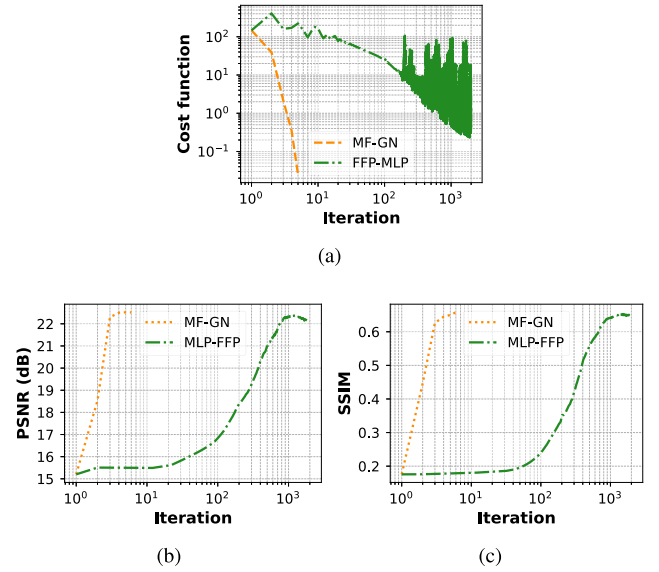


Fig. 7. Performance metrics of MF-GN and FFP-MLP for Experiment 2. (a) Cost function, (b) PSNR, and (c) SSIM. PSNR and SSIM values represent averages computed separately for the reconstructed real and imaginary components of the complex-valued conductivity.

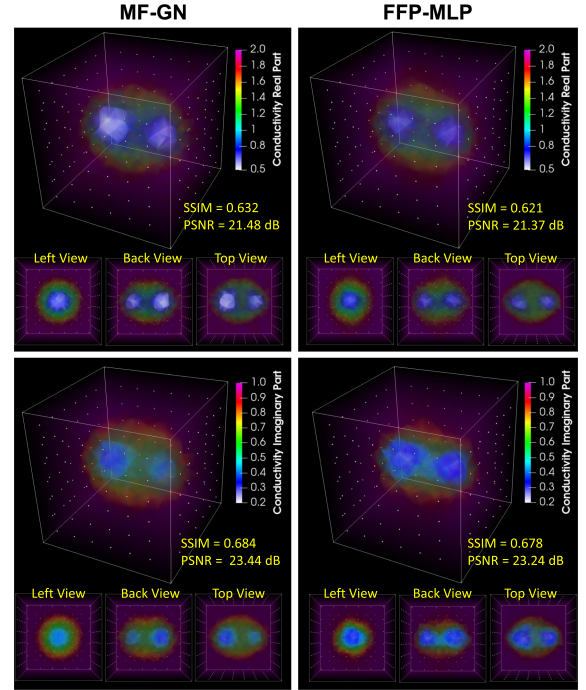


Fig. 8. Reconstruction results of MF-GN and FFP-MLP for Experiment 2. The key parameters of this experiment are described in Section III-B. The MF-GN result is shown after 5 iterations, while the FFP-MLP result corresponds to the 1298<sup>th</sup> iteration, which achieved the highest PSNR and SSIM values among the 2000 iterations evaluated.

mismatch introduced by this interpolation may become problematic when the geometry of the finite element mesh deviates significantly from a rectangular or cubic domain.

The second approach [23], referred to as FFP-MLP in this paper, employs a multilayer perceptron (MLP) with input  $\mathbf{h}$  defined as the Fourier feature projections (FFP) of the finite

element coordinates:

$$\mathcal{FFP}(\mathbf{x}) = \begin{bmatrix} \sin(2\pi\mathbf{B}\mathbf{x}) \\ \cos(2\pi\mathbf{B}\mathbf{x}) \end{bmatrix}, \quad (38)$$

where  $\mathbf{x} = (x, y, z)$  denotes the coordinates of the finite-element centers (tetrahedrons),  $\mathbf{B} \in \mathbb{R}^{n \times 3}$  is sampled from a normal distribution  $\mathcal{N}(0, b^2)$ .  $n$  and  $b$  represent the number and bandwidth of Fourier features, respectively, which must be tuned for specific applications. The use of Fourier features mitigates the spectral bias inherent in standard MLPs and enables the recovery of higher-frequency information.

The third approach, known as GraphEIT [24], adopts a graph neural network (GNN) architecture with the same input of FFP-MLP. This method retains the spectral advantages of Fourier features while allowing the NN to act directly on the graph structure of the finite element mesh.

Among above three NN-regularized methods, FFP-MLP is preferred by us because its simple architecture facilitates hyperparameter tuning, and prior studies have shown its accuracy to be comparable to other NN-based approaches.

To compare MF-GN with FFP-MLP, we implemented FFP-MLP for the small-scale Experiment 2 introduced in Section II-B. In this implementation, the number of Fourier features was set to  $n = 256$  with a bandwidth of  $b = 150$ . The MLP consists of 5 fully connected linear layer, each containing 256 neurons, followed by an output layer with two channels. Each channel outputs a vector whose length matches the number of FEM elements, representing the real and imaginary parts of the FEM-discretized conductivity model. The activate function is chosen as LeakyReLU with the negative slope set as 0.01.

Before solving (36), a pretraining process is implemented to solve the following optimization problem:

$$\theta_0 = \arg \min_{\theta} \|\mathcal{N}(\mathbf{h}, \theta) - \gamma_0\|^2, \quad (39)$$

where  $\theta_0$  is the initial NN parameter for solving (36),  $\gamma_0$  is the initial conductivity model.

Fig. 8 shows the reconstruction results of MF-GN and FFP-MLP, while Fig. 7 presents the corresponding performance metrics across iterations. Based on both the reconstructed images and the PSNR/SSIM values, FFP-MLP is slightly less accurate than MF-GN under the current configuration. We also tested a more conservative step strategy with an increased number of iterations for FFP-MLP. Although this setting produced a smoother convergence curve and a lower final cost function, it led to inferior PSNR and SSIM compared to the results reported here. Nonetheless, given its flexible architecture, FFP-MLP may achieve reconstruction quality beyond MF-GN with dedicated hyperparameter tuning.

Although NN-based regularization methods show promise in improving EIT reconstruction accuracy, their performance is highly sensitive to hyperparameters (e.g., network depth, width, activation functions, dropout rates). As hyperparameter selection lacks theoretical guidance and is typically performed through trial and error, the process becomes computationally prohibitive for large-scale EIT problems. Consequently, a comparison between MF-GN and FFP-MLP is not included for the Experiment 6 that was introduced in Section III-B, as the

inverse problem is too large; a single implementation of FFP-MLP requires tens of hours, making hyperparameter tuning impractical.

## REFERENCES

- [1] M. Cheney, D. Isaacson, and J. C. Newell, "Electrical impedance tomography," *SIAM Rev.*, vol. 41, no. 1, pp. 85–101, 1999, doi: [10.1137/S0036144598333613](https://doi.org/10.1137/S0036144598333613).
- [2] K. F. Morcelles and P. Bertemes-Filho, "Hardware for cell culture electrical impedance tomography: A critical review," *Rev. Sci. Instrum.*, vol. 92, no. 10, 2021, Art. no. 104704, doi: [10.1063/5.0053707](https://doi.org/10.1063/5.0053707).
- [3] N. Polydorides and A. Borsic, "Electrical impedance tomography for geophysical applications," in *Proc. 23rd EEGS Symp. Appl. Geophys. Eng. Environ. Problems*, 2010, pp. 950–954, doi: [10.4133/1.3445538](https://doi.org/10.4133/1.3445538).
- [4] M. Khalighi, B. Vosoughi, M. Vahdat, M. Mortazavi, W. Hy, and M. Soleimani, "Practical design of low-cost instrumentation for industrial electrical impedance tomography (EIT)," in *Proc. IEEE Int. Instrum. Meas. Technol. Conf. Proc.*, 2012, pp. 1259–1263.
- [5] M. Wang, J. Zhao, S. Zhang, and G. Wang, "Electrical impedance tomography based on filter back projection improved by means method," in *Proc. 3rd Int. Conf. Biomed. Eng. Inform.*, 2010, pp. 218–221.
- [6] J. L. Mueller and S. Siltanen, "The D-bar method for electrical impedance tomography—demystified," *Inverse Problems*, vol. 36, no. 9, Aug. 2020, Art. no. 093001, doi: [10.1088/1361-6420/aba2f5](https://doi.org/10.1088/1361-6420/aba2f5).
- [7] B. Zhao, H. Wang, X. Chen, X. Shi, and W. Yang, "Linearized solution to electrical impedance tomography based on the Schur conjugate gradient method," *Meas. Sci. Technol.*, vol. 18, no. 11, Sep. 2007, Art. no. 3373, doi: [10.1088/0957-0233/18/11/017](https://doi.org/10.1088/0957-0233/18/11/017).
- [8] I. Soumaya, L. Mohamed, L. Abdellah, and B. R. Soukaina, "Comparison of iteratively regularized Gauss-Newton method with Adam optimization for image reconstruction in electrical impedance tomography," *Commun. Math. Biol. Neurosci.*, vol. 2023, 2023, Art. no. 128.
- [9] J. Deng et al., "Imaging algorithm and optimal measurement protocol for 3-D impedance tomography on 2-D high-density microelectrode array," *IEEE Sensors J.*, vol. 24, no. 15, pp. 24452–24465, Aug. 2024.
- [10] N. Polydorides and W. Lionheart, "Adjoint formulations in impedance imaging," in *Proc. 3rd World Congr. Ind. Process Tomogr.*, Rockies, Canada, Sep. 2003, pp. p689–694.
- [11] M. R. Islam and M. A. Kiber, "Electrical impedance tomography imaging using Gauss-Newton algorithm," in *Proc. Int. Conf. Inform., Electron. Vis.*, 2014, pp. 1–4.
- [12] M. S. Campisi, C. Barbre, A. Chola, G. Cunningham, V. Woods, and J. Viventi, "Breast cancer detection using high-density flexible electrode arrays and electrical impedance tomography," in *Proc. 36th Annu. Int. Conf. IEEE Eng. Med. Biol. Soc.*, 2014, pp. 1131–1134.
- [13] J. Deng et al., "In-depth imaging of in-vitro tissue models using electrical impedance tomography," in *Proc. 4th Conf. Ind. Computed Tomogr.*, Antwerp, Belgium, Feb. 2025, pp. 689–694.
- [14] A. Denker et al., "Chapter 3 - deep learning based reconstruction methods for electrical impedance tomography," in *Machine Learning Solutions for Inverse Problems: Part A*, Ser. Handbook of Numerical Analysis, A. Hauptmann, B. Jin, and C.-B. Schönlieb, Eds., vol. 26, Elsevier, 2025, pp. 127–175. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S1570865925000031>
- [15] Y. Sun, "Electrical impedance tomography in medical imaging: Fundamental principles, clinical applications, and future innovation trajectories," in *Proc. 3rd Int. Conf. Image, Algorithms, Artif. Intell.*, 2025, pp. 570–579, doi: [10.2991/978-94-6463-823-3\\_57](https://doi.org/10.2991/978-94-6463-823-3_57).
- [16] J. K. Seo, K. C. Kim, A. Jargal, K. Lee, and B. Harrach, "A learning-based method for solving ill-posed nonlinear inverse problems: A simulation study of lung EIT," *SIAM J. Imag. Sci.*, vol. 12, no. 3, pp. 1275–1295, 2019, doi: [10.1137/18M1222600](https://doi.org/10.1137/18M1222600).
- [17] C. Tan, S. Lv, F. Dong, and M. Takei, "Image reconstruction based on convolutional neural network for electrical resistance tomography," *IEEE Sensors J.*, vol. 19, no. 1, pp. 196–204, Jan. 2019.
- [18] Y. Fan and L. Ying, "Solving electrical impedance tomography with deep learning," *J. Comput. Phys.*, vol. 404, Mar. 2020, Art. no. 109119, doi: [10.1016/j.jcp.2019.109119](https://doi.org/10.1016/j.jcp.2019.109119).
- [19] X. Li et al., "An image reconstruction framework based on deep neural network for electrical impedance tomography," in *Proc. IEEE Int. Conf. Image Process.*, 2017, pp. 3585–3589.
- [20] Z. Lin et al., "Feature-based inversion using variational autoencoder for electrical impedance tomography," *IEEE Trans. Instrum. Meas.*, vol. 71, 2022, Art. no. 4504712.

- [21] S. Dittmer, T. Kluth, P. Maass, and D. O. Bager, "Regularization by architecture: A deep prior approach for inverse problems," *J. Math. Imag. Vis.*, vol. 62, pp. 456–470, 2018. [Online]. Available: <https://api.semanticscholar.org/>
- [22] D. Liu, J. Wang, Q. Shan, D. Smyl, J. Deng, and J. Du, "DeepEIT: Deep image prior enabled electrical impedance tomography," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 45, no. 8, pp. 9627–9638, Aug. 2023.
- [23] J. Wang, Y. Wang, J. Deng, and D. Liu, "Unsupervised coordinate-based neural network for electrical impedance tomography," *IEEE Trans. Comput. Imag.*, vol. 9, pp. 1213–1225, 2023.
- [24] Z. Liu, J. Wang, Q. Shan, and D. Liu, "GraphEIT: Unsupervised graph neural networks for electrical impedance tomography," *IEEE Trans. Comput. Imag.*, vol. 10, pp. 1559–1570, 2024.
- [25] D. Knoll and D. Keyes, "Jacobian-free Newton–Krylov methods: A survey of approaches and applications," *J. Comput. Phys.*, vol. 193, no. 2, pp. 357–397, 2004. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S0021999103004340>
- [26] E. J. de de Araujo Jorge Filho and Z. J. Wang, "A matrix-free GMRES algorithm on GPU clusters for implicit large eddy simulation," *AIAA Scitech Forum*, 2021, doi: [10.2514/6.2021-1837](https://doi.org/10.2514/6.2021-1837).
- [27] N. Six, J. Renders, J. Sijbers, and J. D. Beenhouwer, "Gauss-Newton-Krylov for reconstruction of polychromatic X-ray CT images," *IEEE Trans. Comput. Imag.*, vol. 7, pp. 1304–1313, 2021.
- [28] S. Pal, S. Dutta, and R. Maitra, "Fast matrix-free methods for model-based personalized synthetic MR imaging," *J. Comput. Graphical Statist.*, vol. 33, pp. 1109–1117, 2021. [Online]. Available: <https://api.semanticscholar.org/>
- [29] M. HANKE, B. HARRACH, and N. HYVÖNEN, "Justification of point electrode models in electrical impedance tomography," *Math. Models Methods Appl. Sci.*, vol. 21, no. 06, pp. 1395–1413, 2011, doi: [10.1142/S0218202511005362](https://doi.org/10.1142/S0218202511005362).
- [30] N. Hyvönen, "Complete electrode model of electrical impedance tomography: Approximation properties and characterization of inclusions," *SIAM J. Appl. Math.*, vol. 64, no. 3, pp. 902–931, 2004. [Online]. Available: <http://www.jstor.org/stable/4096017>
- [31] B. Jin, Y. Xu, and J. Zou, "A convergent adaptive finite element method for electrical impedance tomography," *IMA J. Numer. Anal.*, vol. 37, no. 3, pp. 1520–1550, 2016, doi: [10.1093/imanum/drw045](https://doi.org/10.1093/imanum/drw045).
- [32] N. Polydorides and W. R. B. Lionheart, "A Matlab toolkit for three-dimensional electrical impedance tomography: A contribution to the electrical impedance and diffuse optical reconstruction software project," *Meas. Sci. Technol.*, vol. 13, no. 12, Nov. 2002, Art. no. 1871, doi: [10.1088/0957-0233/13/12/310](https://doi.org/10.1088/0957-0233/13/12/310).
- [33] A. Adler and W. R. B. Lionheart, "Uses and abuses of EIDORS: An extensible software base for EIT," *Physiol. Meas.*, vol. 27, no. 5, pp. S25–S42, Apr. 2006, doi: [10.1088/0967-3334/27/5/S03](https://doi.org/10.1088/0967-3334/27/5/S03).
- [34] F. Margotti and A. Rieder, "An inexact Newton regularization in Banach spaces based on the nonstationary iterated Tikhonov method," *J. Inverse Ill-Posed Problems*, vol. 23, no. 4, pp. 373–392, 2015, doi: [10.1515/jip-2014-0035](https://doi.org/10.1515/jip-2014-0035).
- [35] A. Lechleiter and A. Rieder, "Newton regularizations for impedance tomography: A numerical study," *Inverse Problems*, vol. 22, no. 6, Sep. 2006, Art. no. 1967, doi: [10.1088/0266-5611/22/6/004](https://doi.org/10.1088/0266-5611/22/6/004).
- [36] M. Kryshchuk and J. Lavendels, "Iterative method for solving a system of linear equations," in *Proc. Procedia Comput. Sci.*, Latvia: iCTE, Riga Technical University, 2016, vol. 104, pp. 133–137. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S1877050917300868>
- [37] Y.-F. Jing and T.-Z. Huang, "On a new iterative method for solving linear systems and comparison results," *J. Comput. Appl. Math.*, vol. 220, no. 1, pp. 74–84, 2008. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S0377042707000417>
- [38] D.-L. Fong and M. Saunders, "LSMR: An iterative algorithm for sparse least-squares problems," *SIAM J. Sci. Comput.*, vol. 33, no. 5, pp. 2950–2971, 2011, doi: [10.1137/10079687X](https://doi.org/10.1137/10079687X).
- [39] F. Toutounian, D. Khojasteh Salkuyeh, and M. Mojarrah, "LSMR iterative method for general coupled matrix equations," *J. Appl. Math.*, vol. 2015, no. 1, 2015, Art. no. 562529, doi: [10.1155/2015/562529](https://doi.org/10.1155/2015/562529).
- [40] Z. Jia, "Regularization properties of Krylov iterative solvers CGME and LSMR for linear discrete ill-posed problems with an application to truncated randomized SVDs," *Numer. Algorithms*, vol. 85, pp. 1281–1310, 2018. [Online]. Available: <https://api.semanticscholar.org/>
- [41] M. Noel and B. Xu, "Archaeological investigation by electrical resistivity tomography: A preliminary study," *Geophys. J. Int.*, vol. 107, no. 1, pp. 95–102, 1991, doi: [10.1111/j.1365-246X.1991.tb01159.x](https://doi.org/10.1111/j.1365-246X.1991.tb01159.x).
- [42] B. H. Brown and A. D. Seagar, "The sheffield data collection system," *Clin. Phys. Physiol. Meas.*, vol. 8, no. 4A, pp. 91–97, Nov. 1987, doi: [10.1088/0143-0815/8/4A/012](https://doi.org/10.1088/0143-0815/8/4A/012).
- [43] C. Dang, C. Bellis, M. Darnajou, G. Ricciardi, S. Mylvaganam, and S. Bourennane, "Practical comparisons of EIT excitation protocols with applications in high-contrast imaging," *Meas. Sci. Technol.*, vol. 32, no. 8, May 2021, Art. no. 085110, doi: [10.1088/1361-6501/abe742](https://doi.org/10.1088/1361-6501/abe742).
- [44] C. C. Onsager et al., "Sensitivity volume as figure-of-merit for maximizing data importance in electrical impedance tomography," *Physiol. Meas.*, vol. 45, no. 4, Apr. 2024, Art. no. 045004, doi: [10.1088/1361-6579/ad3458](https://doi.org/10.1088/1361-6579/ad3458).
- [45] T. A. Davis, "Fill-reducing orderings," in *Direct Methods for Sparse Linear Systems*. New Delhi, India: SIAM, 2006, ch. 7, pp. 99–134, doi: [10.1137/1.9780898718881.ch7](https://doi.org/10.1137/1.9780898718881.ch7).
- [46] A. George and J. W. Liu, *Computer Solution of Large Sparse Positive Definite System*. Upper Saddle River, NJ, USA: Prentice Hall, 1981.
- [47] B. Grychtol, G. Elke, P. Meybohm, N. Weiler, I. Frerichs, and A. Adler, "Functional validation and comparison framework for EIT lung imaging," *PLoS One*, vol. 9, no. 8, pp. 1–13, 2014, doi: [10.1371/journal.pone.0103045](https://doi.org/10.1371/journal.pone.0103045).
- [48] C. R. Vogel and M. E. Oman, "Iterative methods for total variation denoising," *SIAM J. Sci. Comput.*, vol. 17, no. 1, pp. 227–238, 1996, doi: [10.1137/0917016](https://doi.org/10.1137/0917016).
- [49] Z. Chen, Y. Yang, J. Jia, and P. Bagnaninchi, "Deep learning based cell imaging with electrical impedance tomography," in *Proc. IEEE Int. Instrum. Meas. Technol. Conf.*, 2020, pp. 1–6.
- [50] D. Maciejewski, Z. Putowski, M. Czok, and Łukasz J. Krzych, "Electrical impedance tomography as a tool for monitoring mechanical ventilation. An introduction to the technique," *Adv. Med. Sci.*, vol. 66, no. 2, pp. 388–395, 2021. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S1896112621000420>
- [51] A. G. Baydin, B. A. Pearlmutter, A. A. Radul, and J. M. Siskind, "Automatic differentiation in machine learning: A survey," *J. Mach. Learn. Res.*, vol. 18, no. 153, pp. 1–43, 2018. [Online]. Available: <http://jmlr.org/papers/v18/17-468.html>
- [52] A. Paszke et al., "Automatic differentiation in Pytorch," 2017. [Online]. Available: <https://api.semanticscholar.org/>
- [53] C. Wang, H. Deng, and D. Liu, "Physics-driven neural compensation for electrical impedance tomography," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 48, no. 3, pp. 3783–3800, Mar. 2026.
- [54] D. Liu, Y. Wu, B. Tong, and J. Deng, "SDEIT: Semantic-driven electrical impedance tomography," *Neural Netw.*, vol. 197, 2026, Art. no. 108492. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S089360825013735>
- [55] D. O. Bager, J. Leuschner, and M. Schmidt, "Computed tomography reconstruction using deep image prior and learned reconstruction methods," *Inverse Problems*, vol. 36, no. 9, Sep. 2020, Art. no. 094004, doi: [10.1088/1361-6420/aba415](https://doi.org/10.1088/1361-6420/aba415).



**Jian Deng** received the M.Sc. and Ph.D. degrees in geophysics from the China University of Geosciences, Wuhan, China, in 2016 and 2021, respectively. From 2019 to 2021, he was a Visiting Ph.D. Student with Institut de Planétologie et d'Astrophysique de Grenoble, Université Grenoble Alpes, France. He is currently working on super resolution electrical impedance tomography based on micro-electrode arrays as a Senior Researcher with Imec-Vision Lab, University of Antwerp, Antwerp, Belgium. His research interests include tomography techniques, radar tomography, electrical impedance tomography, X-ray tomography, seismic, and acoustic tomography.



**Babu Linkoon P. Meenaketan** received the Ph.D. degree in electromagnetic inverse scattering from BIT-Mesra, Ranchi, India, in 2021. From 2021 to 2022, he held academic positions with JNTUK, India, and IIIT Bhubaneswar, India, from 2015 to 2018. From 2010 to 2012, he was an OSS Engineer with Nokia Siemens Networks. From 2021 to 2024, he was a Postdoctoral Researcher with IMEC, Leuven, Belgium, where he focused on system development for 3D micro-electro-impedance-tomography for cell culture applications. He is currently a Research and Development Engineer with IMEC, where he is involved in the development of scientific systems for organ-on-chip applications. His research interests include tomography imaging, inverse problems, and computational electromagnetism.

**Dries Braeken**, photograph and biography not available at the time of publication.

**Yoke Chin Chai**, photograph and biography not available at the time of publication.

**Tom Torfs**, photograph and biography not available at the time of publication.



**Jan De Beenhouwer** (Senior Member, IEEE) received the M.Sc. degree in computer science engineering from KU Leuven, Leuven, Belgium, in 2003, and the Ph.D. degree in biomedical engineering from the University of Ghent, Ghent, Belgium, in 2008. He was a Postdoctoral Fellow for two years with the University of Antwerp, Antwerp, Belgium, prior to joining the Vision Lab with the University of Antwerp. Since 2018, he has been a Research Professor and currently Co-leads the ASTRA Group, IMEC-Vision Lab which focuses on the development of advanced

computational methods for tomography and new reconstruction techniques that lead to better reconstruction quality compared to classical reconstruction methods. His interests include image reconstruction, processing, and analysis with focus on X-ray computed tomography.



**Jan Sijbers** (Senior Member, IEEE) received the M.Sc. degree in physics and the Ph.D. degree in sciences/physics from the University of Antwerp, Antwerp, Belgium, in 1993 and 1998, respectively. From 2002 to 2008, he was a Postdoctoral Fellow with the Fund for Scientific Research jointly with the University of Antwerp and Delft University of Technology, The Netherlands. Since 2014, he has been a Full Professor with the University of Antwerp and Vice-Chair of the Department of Physics. He is currently a Spokesperson of imec-Vision Lab and

Co-founder of IcoMetrix and Deltaray. His interests include biomedical image reconstruction, processing, and analysis, with a focus on quantitative MRI and X-ray computed tomography. He is a Senior Area Editor of IEEE TRANSACTIONS ON IMAGE PROCESSING and Associate Editor for IEEE TRANSACTIONS ON COMPUTATIONAL IMAGING.